TEACHING OF STATISTICS

The following four papers on the teaching of statistics were presented at a session sponsored by Mu Sigma Rho and the Section on Statistical Education at the annual ASA meeting in San Diego in 1978. The four authors are well qualified to discuss the subject of teaching statistics, and the papers present much helpful material for use by teachers of statistics. Since the four papers contain many personal views and opinions, the Editorial Board felt that it would be most appropriate to publish the papers essentially as they were presented at the San Diego meeting.

Editor

Classroom and Platform Performance

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By discussing some features of elementary and intermediate courses in applied statistics, I plan to bring out a few ideas that I have found useful in teaching. Although no two teachers are alike, I have often found devices used by others adaptable to my own style, and perhaps some of you will resonate to one or two of my remarks. I shall speak as if to new teachers eager for my advice. Some of these remarks also apply to talks given away from home.

When I think of teaching a class, I think of five main components, not all ordinarily used in one lecture. They are

1. Large-scale application
2. Physical demonstration
3. Small-scale application (specific)
4. Statistical or probabilistic principle
5. Proof or plausibility argument.

1. LARGE-SCALE APPLICATION

Only rarely do mathematics and statistics courses give their students views of large-scale applications of the work they study. One may come away from an algebra or a statistics course with a feeling of having collected many tiny clever ideas that work themselves into a major bundle, but without any feeling that the work may have important applications. Nevertheless, students like to know that their studies do relate to important activities in the practical world. By the time students get to college, they tire of being put off with the advice that "You will see applications of this later." Some realize that if they do not see the applications soon, they never will, at least not in formal courses.

Teachers are often reluctant to spend class time on these applications because, of course, the student is not likely ever to be directly involved in such work, and much time is needed to clarify the routine class material that the teacher feels obligated to deliver. We cannot fault this view very much but maybe a little.

Until recently, few teachers had a collection of substantial uses of statistics at hand unless they had worked in many projects or had spent a great deal of time collecting such examples and writing them up. Today this is easier because we have the book *Statistics: A Guide to the Unknown* (Tanur et al. 1978), which has 46 essays (in the 1978 second edition) on important uses of statistical methods in many practical areas. This book was prepared for the Joint Committee on Statistics and Probability of the American Statistical Association and the National Council of Teachers of Mathematics. (The American Statistical Association receives the royalties and uses part of them to support its educational efforts.)

These essays have been so popular that the publisher has broken the book into three parts and published paperbacks in the business and economics areas, in the political and social areas, and in the biomedical and health areas (Tanur et al. 1976, 1977a,b). In the first edition, these essays had no accompanying discussion questions, but these have been added owing to the need perceived by Erich Lehmann, the publisher's editor, and so both the small specialized paperbacks and the second edition of the full work have problem material.

My main point is that with these essays it is much easier for the teaching statistician to open a lecture with a few remarks about some real-world problem that uses the specific method to be treated in the day's lecture. I do not begrudge a little time thus spent. In the long run, most of these students will not actually use statistical methods in detail, and what they may get out of the course is the idea of the important or intriguing applications. Furthermore, one never knows how a student is listening. All teachers are, I am sure, astonished again and again, as I have been, with what sticks in a student's mind about a course.

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A second resource of practical problems is the four volumes of *Statistics by Example* (Mosteller et al. 1973). The fourth is especially oriented to statistical modeling.

For content, then, I would say that in many elementary courses it may be possible and profitable to start many of the lectures with some remarks about a problem in which the statistics of the lesson plays a role, even though a minor one.

2. PHYSICAL DEMONSTRATION

Gathering relevant data in class and then analyzing them with the help of the class gets the students involved because we are analyzing their data and they can contribute to the analysis.

One way to obtain a set of data that can be intriguing is to collect for males and for females, separately, the number of brothers and the number of sisters each student has. One has to specify the degree of relationship, or the matter can get very complicated. I stick to brothers and sisters who have the same biological father and mother as the respondent. Usually the men have more sisters than the women have, and the women have more brothers than the men. Sorting out why this is can be instructive.

But the data can also be generated by the students themselves. We can look at guessing sequences. For example, one can ask each student to act like a random-number table, except that the student chooses one of the numbers 1, 2, or 3 with replacement after a signal and writes down the number thought of. Ask the students to close their eyes and as you time them (five seconds is time enough between signals to guess one number), write a sequence of numbers, one every five seconds, on a piece of paper. An occasional student will pick a number and always use it, say 3,3,3,3,3,3,3,3,3,3, I suppose, to show that he or she is not a random-number machine—forgetting that the problem was to what extent the machine could be imitated. In a large class, it is instructive to analyze the numbers of successive pairs of digits generated by the students, skipping past, say, the first three marks to get past the start-up phase.

Or each student can have a die and be asked to cast it and record its outcome 10 times in sequence, and then the data can be collected and analyzed. You need to make rules about what tosses count or don't count (how about not falling flat or falling on the floor, etc.). It is preferable if the students keep their own data, or at least a copy of them, to keep them involved. What should be analyzed? The sequence (tendency of numbers to follow one another and thus see if we have independence, or something close to it). Another grabber is the distribution of the lengths of runs. This can be recovered from the same data because longer runs are found if we redefine the die tosses as odd and even (rather than the numbers from 1 to 6) so that the die can simulate a 50–50 coin.

Many teachers have collections of colored balls with paddles or other devices for drawing the balls out in an essentially random way so that we can observe real-life binomial variation in a somewhat controlled environment.

I wish that every class I taught could have a physical demonstration of these sorts that each student can be involved with, and the more mathematical the class, the greater the need.

Once in a while, a good physical demonstration goes astray. For example, I had a nice set-up once that depended on some cups having numbered tags in them, so that we could carry out a sampling demonstration. The helpers who were passing through the class got mixed up and did not always report the minus signs on numbers that were drawn, and one of the helpers also did not recover the tags after they were drawn and replace them and mix them up. When a demonstration of this kind begins to go wrong in such ways, I have a firm rule: Abandon it for that day. Any attempt to get everything fixed up and back in order will merely destroy the rest of the hour.

A demonstration has to be worked out in all details before one comes to class. The helpers, if there are to be any, have to be poised and already trained. The students need to know what the instructions are. If you think of possible misunderstandings, do not rely on the fact that no one asks a question after you ask for questions. If no one asks, then say, "All right, then I have a few," and then go over the instructions by asking questions about them and calling on specific students to reply. You will likely find that some of the instructions have not been understood, especially if there is anything the least bit tricky. It is good to remember that at any time some people are not paying attention, and at other times, no one is.

Needless to say, we are not trying to shame the students, we are just making sure that the usual sort of Murphy's law (anything that can go wrong will go wrong) has a lower rate of application in your classroom. The teacher who uses physical demonstrations must be prepared for them to foul up and also to take such failures in good humor when they occur. Some students get fun out of a teacher's discomfiture, so be prepared both to be cheerful anyway and not to blame anyone, including yourself.

When I speak of foul-ups, I am talking about dropping 1,000 beads on the floor, or someone's forgetting to write down the numbers in a manner useful for analysis. I am not talking about the possibility that the outcome of the experiment may differ somewhat from theoretical results. Part of the purpose of the investigation is to experience the sort of departure from theory that naturally occurs.

We need to remember that Walter Shewhart, in setting up quality-control methods, regarded such distributions as the Poisson, the binomial, and the normal as ideals to be achieved when all the assignable causes had been removed after considerable engineering work and research. We cannot expect one-shot classroom experiments to have the kind of perfection that industry can achieve only at great expense and effort. Therefore, when the students try to "cheat" by picking big numbers out of a hat or peeking ahead, and so on, please regard these as aberrations from the ideal that you should call attention to, without rancor, and then look forward to seeing whether their effects may show in the analyses.

Today, of course, the interactive computer gives us a chance to do more and better experiments of a stochastic nature than we formerly could. But let us remember that that is all going on in a black box. We still have the question, "Do physical objects behave as they are supposed to according to our ideal theory?" Therefore, some experience with real coins, cards, or dice can be most instructive before the computer takes over. And, indeed, it can be informative to compare the results that are achieved by real objects or real people with the performance of computerized random-number simulations of them. After a small classroom experiment with coins, I have found some of the class interested in William H. Longcor's millions of tosses of dice (Iversen et al. 1971).

3. SMALL-SCALE APPLICATION

In preparing to present a specific technique, we need a concrete real-life problem that uses the technique. This is not quite the same as the large-scale application in which the technique is mentioned under Section 1; instead, it is the specific use of the technique, often complete with numbers.

Ideally, the material is available on a handout, which the student either has from the beginning of the semester in a syllabus or workbook, or it may be handed out on the day of the lesson. Fresh handouts revive the student, if their number is not overwhelming.

This concrete example makes sure that the student understands what the specific technique is supposed to accomplish and that the technique has a practical use. These applications are best drawn from the areas of the students' interests, because students like to learn their own subject matter at the same time that they are learning methods and techniques.

THE LESSON:
4. STATISTICAL OR PROBABILISTIC PRINCIPLE
5. PROOF OR PLAUSIBILITY ARGUMENT

You may be thinking, perhaps, "Between the big illustration, the small illustration, and the demonstration, we are never going to get to the lesson," but here we are with it. I have very little to say about it today except that a friend of mine, Robert E.K. Rourke, once taught me an important idea for presenting new material. He calls it PGP. These letters stand for

Particular General Particular.

In our lesson strategy, the small illustration (Section 3) is the first *particular*. It motivates the lesson and clarifies the technique.

The *general* is the general treatment of the technique that you present. Let us say it is a treatment of the unbiased estimate of the variance of a distribution based on a random sample. We will display the general formula and, depending on the class, prove unbiasedness, or discuss and motivate the $N - 1$ in the denominator, where N is the sample size, paying special attention, say, to samples of size 1 and 2. At any rate, when this is completed, we will have handled the general part of the discussion.

Now Rourke's idea is to drive the whole thing home with a second *particular*, that is, with a new and further use of the method. The second particular could be, to a new example, similar to the old one, followed by a slightly different one, perhaps with some extra fillip. For example, if we have been looking at the variability of single measurements, we might apply the same technique to the variability of the sum of two independent measurements to see how the results compared with those for single observations.

In a mathematically oriented class, I do not regard it as satisfactory merely to prove a theorem and apply it. I like to spend some time, whenever I can, seeing if we, the teacher and the class, can make the whole thing seem reasonable as well, often qualitatively. In the problem of variability just mentioned, we ought to think about whether the variability of a sum is going to be larger or smaller than that of the single measurements, in intuitive terms as well as formal mathematical ones. If I have plenty of time during the lesson I like to work in a discussion of the law of sums and the law of averages simultaneously here.

At any rate, such material would complete the PGP principle for the lesson.

All told, then, when I prepare a lesson, I think of five main kinds of content in a statistics or probability lesson, the five previously mentioned. I rarely find it possible to get all five in, I am sorry to say, but I can usually do four.

To do this requires a good deal of planning and sometimes timing. Let me turn to a few remarks about lesson preparation.

Lesson Preparation. When trying for a good demonstration, either a physical one or a mathematical one, the teacher needs a clear notion of how long it is going to take and also needs to know what language to use. In some demonstrations, certain sentences, when developed just right, are very easy to understand and they carry the message. But the same ideas, when started out a little wrong, never seem to come to an end, except possibly the dreaded "You know what I mean even if I can't say it." If I wind up there, I can then be sure that students don't know what I mean. Thus, memorizing starts of a few key sentences in a lecture can make smooth going. Definitions and specific explanations often need this special treatment.

I find that when I am going to give a new demonstration, I want to sit down and tell it silently to myself because I am very comfortable and cozy that way, and I have my notes, and I don't have to say it out loud. Unfortunately, this sort of comfort is almost sure to lead to trouble later. What the teacher needs to do is to lay down the notes, get chalk, write down the time, and begin the demonstration out loud, writing on the board as planned. This brings out the embarrassing tough spots. When the rehearser doesn't know what comes next, long silences or much "ah-ahing" goes on while the clock ticks on. Furthermore, there is not enough board space in the world to put down all that has been planned. And when the rehearser comes to the end of a "five minute" demonstration and finds that half an hour has gone by, he or she is likely to believe that in the class it will all go much more quickly. It won't. It will go more slowly because someone will ask a tough question and derail the instructor right at a key point.

We need practice, and we need to figure out just what equations are to appear on the board and what example is to be used. We must time the example as well, especially if it has any calculation, algebraic or arithmetic. Again, however clever we are, it is well to have the example completely worked out and to know exactly how it goes. Even then, once in a while we may find that the results found in class are not identical with what was worked out at home.

I realize that some people can use what I call the stumblebum technique effectively. They start out on an example, and seem to make mistakes and get the students caught up in the demonstration helping them. I once admired an elementary school teacher who had 50 mathematicians vying to correct him as he kept apologizing for not knowing as much as they did. I suddenly realized that he was egging them on the same way he did the children. It was great. I've used his actual lecture to teach the idea of the distribution of the sum of two random variables, but not the stumblebum technique. This is a most effective device, but I believe that it can be used by few teachers and by these only when they have complete command of the material and total self-confidence.

I would not recommend memorizing talks, although that is what some teachers do, but it is well to be so familiar with a talk that one can concentrate on how to say it rather than on what it is that will be said.

I would like to summarize this point by saying that rehearsals are extremely helpful, and rehearsals with timing very instructive. Rehearsals are, I think, the single best way of improving one's lecture work. Backup in the form of notes in big, black block letters can also be most helpful.

Length. When one finds that the talk is too long, one's first impulse is to plan to speak faster. This is fatal. The basic rule should be that when a lecture or a talk is too long, we have to abandon whole chunks: a demonstration, a proof, an example, and so on. The lesson plan would ideally have an arrangement something like a news story that is designed still to be informative when cut off at the close of any paragraph. The intent is to do thoroughly what is done. As George Miller says, "The job is not to *cover* material but to uncover it." Thus, it is good to have a lesson in which the main body of the material is delivered by about two-thirds the way through the class period, and the rest of the lesson plan amplifies through examples and special cases or more esoteric ideas, as far as time is available.

I think it well to recognize that students remember very little of what is said in the last couple of minutes before the bell rings, and I doubt if anything remains of what is said after the bell rings, unless possibly it is an example said to be on the final examination.

Shortness. A related matter does occasionally arise when a teacher fears not having enough material to fill up the time. Then there is likely to be an opening with anecdotes and filler material before getting into the heart of the material. Inevitably, toward the end of the lecture, the teacher apologizes by saying that there is a lot of interesting material that there is not time to tell. Visiting speakers to a class are especially prone to this sort of thing. The better students become very restless.

If I think there isn't enough material for the lesson, and I have no way to get more prepared in time, I start right in with the best material as if there was not enough time, and deliver it at the regular rate, and then take advantage of the leftover time for discussion. Usually some difficulty arises that uses the time profitably.

Handwriting. In spite of the 20th-century visual aids, blackboard and chalk are still a mainstay for most of us. I see nothing wrong with this. Some people, like me, have rather poor handwriting, and to them I say just one thing, write large. This helps a great deal.

Then there are those who write beautifully and small, and very lightly and swiftly, so that students can't keep up. I assume that there is a special part of the Inferno waiting for them, but I cannot imagine a sufficiently severe punishment. For those who stand in front of what they have written and block it, no doubt the afterlife holds still worse.

Projectors. The overhead projector has brought us a long way. I have recently been subjected, however, to a great many lectures given by people using transparencies. There are a few features of working with the projector that I would like to call to your attention.

The figure is a typical transparency. (The speaker displayed a transparency produced by typing from a manuscript, single spaced, at full 8½ inch width after reduction, 62 lines long.) It has many advantages: it has everything on it, all the formulas, all the lecture, and nobody can read it, except possibly the lecturer, who then bores the audience to tears by doing so.

I think the rule most often broken in using the overhead projector is that material for it should be written large and with few lines to the page. Using several transparencies instead of one is all right. It may some-

14B. Matching as a Way of Fitting

Given a variety of carriers, $x_1, x_2, \ldots, x_k$, the x's now being different variables, no two identical, each taking on a set of values, let us think of fitting $\beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k$, a linear multiple regression on our carriers. Some of the x_i may be functions of others. One of the x's may be a constant; for example, x_1 will often have the single value 1 for all data sets.

To get values $\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_n$ we need some equations. Life is simplest when these are linear equations in the $\hat{\beta}$'s. Let $y = \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \ldots + \hat{\beta}_k x_k$ be the fit, once we have reached it.

Let us now lead up to a method of getting simple linear equations for the $\hat{\beta}$'s.

If we have a set of coefficients $\{h(i)\}$(where the letter i corresponds to the ith set of observations, $i = 1,2,\ldots,n$), we will call the set a *matcher*, if and only if it produces the following equality for any data and the corresponding fit $\Sigma h(i)y(i) = \Sigma h(i)\ y\ (i)$. Thus the matcher-weighted sum of the fitted values of the response variable. Every such set of coefficients that satisfies (*) for a specific kind of fit we call a matcher for that fit.

What are some simple examples? When we fit $y = x$ by ordinary least squares, βx is a matcher, for to require $\Sigma x(i)y(i) = \Sigma x(i)\hat{y}(i) = \Sigma x(i)\beta x(i) = \beta\Sigma x(i)x(i)$ is to require $\Sigma x(i)y(i) = \beta\Sigma x(i)x(i)$. This is equivalent to the equation for β we used earlier $\beta = \Sigma xy/\Sigma x^2$.

When we fit $y = \alpha + \beta x$ or, equivalently, $y = \mu + \beta(x - \tilde{x})$, then 1 (a constant), x, and $x - \tilde{x}$ are all matchers. The vorresponding conditions are for $\hat{\mu}$ and β: 1 as matcher: $\Sigma 1 \cdot y(i) = \Sigma 1 \cdot \hat{y}(i)$, $n\tilde{y} = n(\hat{\alpha} + \beta x) = n\hat{\mu}$; x as matcher: $\Sigma x(i)y(i) = \Sigma x(i)\hat{y}(i) = \hat{\alpha}\Sigma x(i) + \beta\Sigma x(i)x(i) = \mu\Sigma x(i) + \beta\Sigma x(i)(x(i) - \tilde{x}); = \hat{\beta}\ \Sigma(x(i) - \tilde{x})^2$ which also give the familiar results for $\hat{\mu}, \hat{\alpha}$, and $\hat{\beta}$.

Some algebra for matchers. Matchers come in bundles. Suppose $h = \{h(i)\}$ and $k = \{k(i)$ are matchers. Then $\Sigma h(i)y(i) = h(i)\hat{y}(i)$, and $\Sigma k(i)y(i) = k(i)\hat{y}(i)$. Adding with various weights gives

A Typical Transparency

(Mosteller, F., and Tukey, J.W. (1977), Data Analysis and Regression, *Reading, Mass.: Addison-Wesley. Reproduced with permission.)*

times be desirable to have two projectors going instead of one.

These machines are easy to use, and the overlays can be easily and quickly prepared even by an amateur. Time and trouble are repaid for good-looking overlays, though. This projector is a great supplement for a handout.

One can write with a grease pencil or a special pen on the overhead projector's overlay instead of on a blackboard, but it takes practice.

The speaker may not realize how much small movements are magnified by the projector. When pointing to something with a pencil, rest the tip on the plastic to stop the quivering, or lay it down. Don't point with fingers, and don't wave hands over the transparency. All of these sudden movements are hard on the viewers' eyes and make some viewers seasick. Aside from crowding, sudden scrubbing and waving movements are the most frequently employed poor form in using transparencies.

Finally, when finished with a transparency, talk about something else for a bit before removing it.

Sometimes one gives a lecture in a special place or plans to use special equipment for the day—a special movie projector, an unusual slide projector, or other equipment. Unless you have actually tried this equipment out, are sure that it will be available when the day comes, and are sure that you can run it or that you will surely have a competent helper, you must be prepared for evasive action when this equipment does not work out.

I once had a special short film that had to be run on an unusual machine. I got the machine and a young man from our audiovisual-aids department, and we spent a couple of hours learning how to run the machine so that it produced exactly the effect desired. I was very pleased. Next morning when the class was to start at 9:00 a.m., the helper did not appear, but rather late someone else appeared, explaining that the other chap had a class and couldn't come. The new fellow couldn't thread the machine in the first hour. One must be prepared for such a disaster and plan to abandon the show and replace it instantly with other relevant material. One can't afford to waste the class's time.

Questions to the Audience. I have intimated and emphasized that it is good to have the students involved. For this reason, even in large lectures, I feel that it is good to have questions for the audience. I might mention that I have a friend who has a trick that I haven't tried yet but that I would like to try. He distributes to his students cubes differently colored on several sides. Then he asks the students multiple-choice questions and tells them that if they prefer one answer to show the red side, another the yellow, and others the green, and so on. This device seems to keep large classes alert, and I pass the idea along for what it is worth. The direct participation of the class has the advantage of keeping all its members involved with the material.

The rest of the paper is oriented more to lectures given on trips to unfamiliar places.

The Handout. People like to take something away from a talk. If you have a handout, you have already made sure that they will. You have also taken out insurance against a major omission in your presentation. You have prepared for amplification in case of questions. You have put the listeners in the position of being able to go at a slightly different pace than you do, and they can go back and check the definitions and conditions in a leisurely manner. You have also shown that you cared enough about the audience to take the trouble to prepare.

Try to find out about how many handouts you will need and take plenty. Do not depend on the mail.

Be sure your handout has

a title
a date
your name and affiliation
reference to foundation or other support, if any
numbered pages.

Some speakers include a reference to the place where the talk is being given. Beyond this, a lot depends on the topic, but

key definitions
theorems
proofs
formulas
references
tables
figures
results

are examples of valuable things for the reader to carry away and for you to lean on in your presentation. On the one hand, you need not give out the whole paper; on the other hand, you may wish to distribute more than you can present.

Variety. In keeping people involved, the keynote is variety. One source of variety is the blackboard, colored chalk, the examples, the lecture, moving around, the projector, the physical demonstration, the questions and answers, and the handout. The changes in method of presentation keep bringing the audience back to the talk. Try not to use just one method of presentation, even if you can. Another source of variety is content, but that is another story.

Hazards. In giving speeches away from home, be prepared for the possibility of poor facilities. Lack of chalk is a common problem, no eraser is another. In some restaurants and hotels, the chalk won't write on the board. If you carry a few pieces of soft chalk to such talks, you may be pleased. If you need colored chalk, carry your own, but be sure to try it out first. Only rather bright-colored chalk shows up. Those beautiful dark purples and blues are useless.

A lecturer with a handout is ready when told "We had no idea this place had no blackboard; they hold conferences here all the time."

Before a talk, go to a bit of extra trouble. Find out where it is to be held. Go there and try everything out. See if you can turn the lights on and off. If there is a projector, be sure you can plug it in and that it works; focus it. In the case of an overhead projector encourage the persons who provide it to provide an extra bulb in case the bulb fails. If it does, don't burn your hand by touching it directly.

If there is audio equipment, try it out. Find out how far you should be from the mike to make the right sort of noise. Have some water nearby in case your throat goes dry. A sip clears up lots of voice troubles for a speaker.

You may feel embarrassed about all these preparations, and anyone testing a mike always feels silly. But you'll look even sillier fumbling around during the talk.

If you haven't enough handouts, get the ushers to distribute one to every second or third person. If you run out, offer to send material to those who give their names and addresses. One can be surprised when a lecture for 30 turns into 300 because some teachers are attracted by the title and bring their classes at the last minute.

Don't distribute the handouts yourself. The speaker has other things to do.

Strange things happen. Be ready for them.

1. The room where the talk is to be given is occupied. Relax. Let the local people work it out. Introduce yourself to people waiting around and chat with them. Don't complain.

2. The room isn't big enough! Already you are a great success. Don't apologize for the hosts. You can say how happy you are to see such a large audience. Don't apologize for late starting because of local arrangements. Only apologize for lateness if it is your fault.

3. Only two or three people show up. Frequently there will then be a dither about whether the speech should be given since there are so few. Indicate that you would like to give the speech and give it, perhaps a little more intimately. Don't talk about the smallness of the audience. I have made some long-term acquaintances this way, for example, on one evening during a 14-inch snowstorm.

However carefully one prepares, time problems can arise not of one's own making. You were promised 45 minutes, but the previous speaker ran over. Everyone thought lunch was at 1:00 p.m., but the caterer says 12:30 p.m. or not at all, and now you have 15 minutes including the question period. What do you do? First, don't complain. Second, the Gettysburg Address didn't take 15 minutes. If you have prepared to dump, as prescribed before, you know exactly what the most important 15 minutes of your talk is. It is 12 minutes long, plus 3 minutes for questions. Don't rush. Carefully tell what is important and what the implications are, and then stop for questions. Whatever you do, don't talk about not having much time. That just upsets the audience, and they start paying attention to how you are managing the talk instead of what you are saying. Finally, don't feel insulted when things like this happen. They happen to everyone everywhere. It is too bad, but just step in and perform. People will be pleased with a nice short speech. I believe that Paul Halmos, a very great lecturer, noted that in a lifetime of giving and attending mathematics lectures he had never heard complaints about a seminar ending early. The classical difficult situation occurs when you are the final discussant and the introducer says, "After the next speaker, we will turn to cocktails, which I am informed are already in the hall." I think you would do well to restrain yourself to about 7 minutes in such circumstances.

Conclusion. I'm sure that there is no one good way to teach, but that there are many, and, unfortunately, still many more ways to do it badly. What a speaker hopes to accomplish in a talk like this is to distribute a few ideas. I once complained about a cookbook to a friend who is a master chef. He said that if one gets one good recipe out of a cookbook, that is the most that one can expect. I hope you have found one thing in my discussion that you can adapt to your teaching. And I shall be most interested if, in the question period, you comment on ways you find useful to think about a lesson.

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The Teaching of Statistics: Content Versus Form

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1. INTRODUCTION

This essay is a write-up of a portion of a panel discussion on the topic. It is closely related but not isomorphic in form and content to what was actually presented. Some aspects are given a somewhat different form and there are some aspects that were not given in the oral presentation because there was not time. I judge it important to include here the abstract I gave beforehand on my presentation, because it presents in succinct form what I regard as key ideas. Also, this has utility because I shall not have space here to discuss all the ideas. These are as follows:

> Aims of the discipline; statistics is not mathematics; the foundations are not in mathematics; mathematics should be the servant of statistics and not the master; ideas and aims must determine the mathematics and not vice versa as is predominantly the case; content should dominate form; the ultimate content must be philosophical; what is probability and what is the role of probability in human affairs? In the beginning was not the "word" but a problem; data analysis without a problem is pure waste; the flooding of humanity with data and data analyses; association or correlation versus causation; mere observation can be totally misleading; the prime thrust must be towards intervention; the dangers of rotten statistics and examples thereof.

Here, in my introduction, I shall give some general reactions to the overall huge topic. I find the following perceptions that I have to be interesting and relevant:

1. Every substantive area raises perennially the question "How to teach X?" We can see that there are no simple answers. Every discipline exhibits a wide variety of reasonable opinion.

2. We saw a revolution in the teaching of mathematics, starting in the mid-fifties, and now we see presentations, especially by M. Cohen, which have some force to most of us, to the effect that that revolution was a huge mistake. The claim is made that not only is it the case that students cannot do simple mathematics, such as the 3R's, or at a slightly sophisticated level, integrate x or 1/x, but also it is the case that professors can't teach. And some surely have similar perceptions about the students and teachers of statistics.

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