

(1- 2α) Confidence limits for the expectation [i.e. the 'mean' parameter] of a Poisson random variable

E.g. if observe 6 events in a certain amount of experience, then 95% CI for the μ count for this same amount of experience is (2.20, 13.06)

1-2		0.998		0.99		0.98		1-2		0.95		0.9		0.8	
		0.001		0.005		0.01				0.025		0.05		0.1	
count	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	count	Lower	Upper	Lower	Upper	Lower	Upper
0	0.00	6.91	0.00	5.30	0.00	4.61	0.00	4.61	0	0.00	3.69	0.00	3.00	0.00	2.30
1	0.00	9.23	0.01	7.43	0.01	6.64	0.01	6.64	1	0.03	5.57	0.05	4.74	0.11	3.89
2	0.05	11.23	0.10	9.27	0.15	8.41	0.15	8.41	2	0.24	7.22	0.36	6.30	0.53	5.32
3	0.19	13.06	0.34	10.98	0.44	10.05	0.44	10.05	3	0.62	8.77	0.82	7.75	1.10	6.68
4	0.43	14.79	0.67	12.59	0.82	11.60	0.82	11.60	4	1.09	10.24	1.37	9.15	1.74	7.99
5	0.74	16.45	1.08	14.15	1.28	13.11	1.28	13.11	5	1.62	11.67	1.97	10.51	2.43	9.27
6	1.11	18.06	1.54	15.66	1.79	14.57	1.79	14.57	6	2.20	13.06	2.61	11.84	3.15	10.53
7	1.52	19.63	2.04	17.13	2.33	16.00	2.33	16.00	7	2.81	14.42	3.29	13.15	3.89	11.77
8	1.97	21.16	2.57	18.58	2.91	17.40	2.91	17.40	8	3.45	15.76	3.98	14.43	4.66	12.99
9	2.45	22.66	3.13	20.00	3.51	18.78	3.51	18.78	9	4.12	17.08	4.70	15.71	5.43	14.21
10	2.96	24.13	3.72	21.40	4.13	20.14	4.13	20.14	10	4.80	18.39	5.43	16.96	6.22	15.41
11	3.49	25.59	4.32	22.78	4.77	21.49	4.77	21.49	11	5.49	19.68	6.17	18.21	7.02	16.60
12	4.04	27.03	4.94	24.14	5.43	22.82	5.43	22.82	12	6.20	20.96	6.92	19.44	7.83	17.78
13	4.61	28.45	5.58	25.50	6.10	24.14	6.10	24.14	13	6.92	22.23	7.69	20.67	8.65	18.96
14	5.20	29.85	6.23	26.84	6.78	25.45	6.78	25.45	14	7.65	23.49	8.46	21.89	9.47	20.13
15	5.79	31.24	6.89	28.16	7.48	26.74	7.48	26.74	15	8.40	24.74	9.25	23.10	10.30	21.29
16	6.41	32.62	7.57	29.48	8.18	28.03	8.18	28.03	16	9.15	25.98	10.04	24.30	11.14	22.45
17	7.03	33.99	8.25	30.79	8.89	29.31	8.89	29.31	17	9.90	27.22	10.83	25.50	11.98	23.61
18	7.66	35.35	8.94	32.09	9.62	30.58	9.62	30.58	18	10.67	28.45	11.63	26.69	12.82	24.76
19	8.31	36.70	9.64	33.38	10.35	31.85	10.35	31.85	19	11.44	29.67	12.44	27.88	13.67	25.90
20	8.96	38.04	10.35	34.67	11.08	33.10	11.08	33.10	20	12.22	30.89	13.25	29.06	14.53	27.05
21	9.62	39.37	11.07	35.95	11.83	34.35	11.83	34.35	21	13.00	32.10	14.07	30.24	15.38	28.18
22	10.29	40.70	11.79	37.22	12.57	35.60	12.57	35.60	22	13.79	33.31	14.89	31.41	16.24	29.32
23	10.96	42.02	12.52	38.48	13.33	36.84	13.33	36.84	23	14.58	34.51	15.72	32.59	17.11	30.45
24	11.65	43.33	13.26	39.74	14.09	38.08	14.09	38.08	24	15.38	35.71	16.55	33.75	17.97	31.58
25	12.34	44.64	14.00	41.00	14.85	39.31	14.85	39.31	25	16.18	36.90	17.38	34.92	18.84	32.71
26	13.03	45.94	14.74	42.25	15.62	40.53	15.62	40.53	26	16.98	38.10	18.22	36.08	19.72	33.84
27	13.73	47.23	15.49	43.50	16.40	41.76	16.40	41.76	27	17.79	39.28	19.06	37.23	20.59	34.96
28	14.44	48.52	16.25	44.74	17.17	42.98	17.17	42.98	28	18.61	40.47	19.90	38.39	21.47	36.08
29	15.15	49.80	17.00	45.98	17.96	44.19	17.96	44.19	29	19.42	41.65	20.75	39.54	22.35	37.20
30	15.87	51.08	17.77	47.21	18.74	45.40	18.74	45.40	30	20.24	42.83	21.59	40.69	23.23	38.32

• Computed from (exact) Poisson tail areas i.e. $\text{Prob}(\text{COUNT} \geq \text{count} \mid \mu_{\text{Lower}}) = \text{Prob}(\leq \text{count} \mid \mu_{\text{Upper}}) = \dots$. See also the spreadsheet "Exact confidence limits on a Poisson parameter" on 626 website • Limits in above Table computed using exact relationship b/w Poisson and Chi-square tail areas (later).

Basis for "First Principles" Poisson Confidence Interval : not your usual " point estimate +/- some multiple of the standarderror"

To form a (1-2) CI for μ , based on # events c , need to find:

$$\mu_{\text{LOWER}} \text{ such that } \text{Prob} (c \text{ or more} \mid \mu_{\text{LOWER}}) =$$

$$\mu_{\text{UPPER}} \text{ such that } \text{Prob} (c \text{ or fewer} \mid \mu_{\text{UPPER}}) =$$

Example: 95% CI based on $c = 6$.

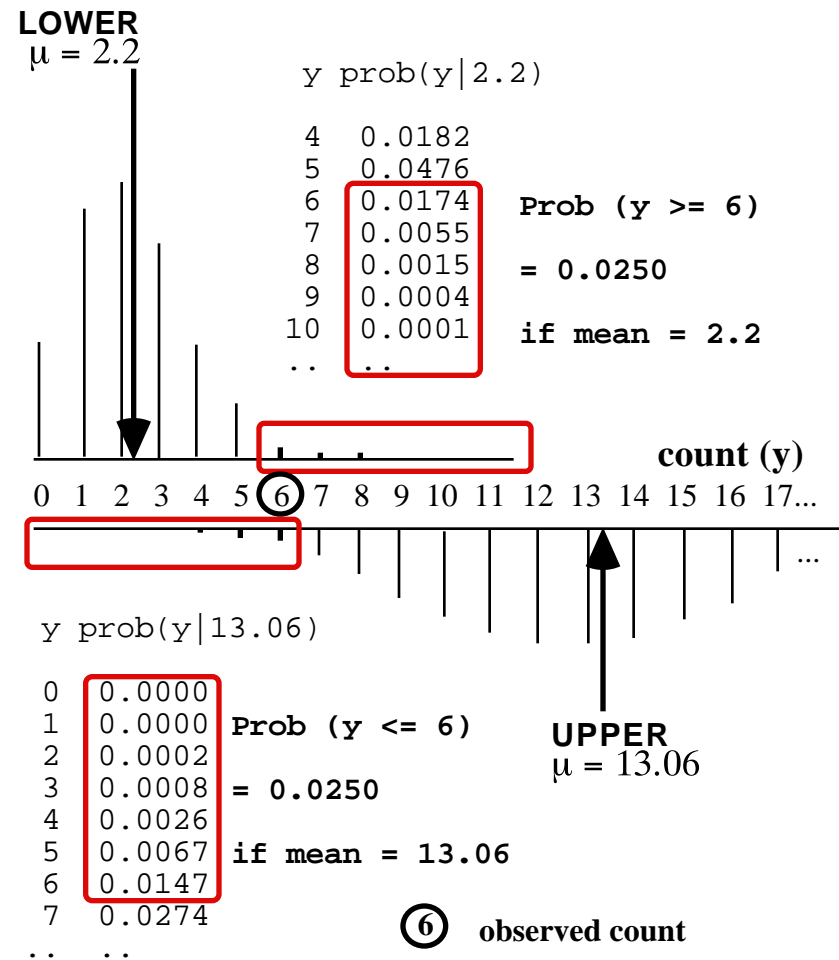
Need to find the μ_{LOWER} that makes the probability of 6 or more equal to $= 0.025$.

Need to find the μ_{UPPER} that makes the probability of 6 or fewer equal to $= 0.025$.

Finding lower and upper limits involves "trial and error" to find the appropriate μ_{LOWER} and μ_{UPPER} that yield the target 's.

See below for a way to get there directly using Link between the tail areas of the Poisson and tail areas of Chi-Square distributions. Note that the above "First Principle" is a general and important one; it "just so happens" that in this particular discrete distribution, if one has access to the percentiles of the Chi-Square distribution, the link helps avoid the "trial and error" process involved in the actual calculation.

Note that unless c is large, the Poisson distribution corresponding to the lower limit will be skewed and not always amenable to a Gaussian approximation; the Poisson distribution at upper limit is less troublesome.



"Exact" CI for mean, μ , of a Poisson distribution using Link between Poisson and Chi-Square tail areas.

This is a surprising link, considering that the Poisson is a distribution on the discrete integers 0, 1, 2, ... and the Chi-square distribution is on the non-negative real numbers.

It has been known since early in 20th century, but has remained largely hidden. (the proof requires integration by parts!)

It used to be an important exact alternative to summing tail areas until the Poisson (and several other) tail areas became available from built-in functions in the statistical and spreadsheet packages. Even now, with a spreadsheet formula for Poisson tail areas, one has to find the limits by trial and error.

To form a $(1-2\alpha)$ CI for μ , based on a count c , we need to find

$$\mu_{\text{LOWER}} \text{ such that } \text{Prob}(c \text{ or more} \mid \mu_{\text{LOWER}}) =$$

$$\mu_{\text{UPPER}} \text{ such that } \text{Prob}(c \text{ or fewer} \mid \mu_{\text{UPPER}}) =$$

Example: Based on $c = 6$, find 95% CI for μ

Need to find the μ_{LOWER} that makes the probability of 6 or more equal to $\alpha = 0.025$. and the μ_{UPPER} that makes the probability of 6 or fewer equal to $1 - \alpha = 0.975$.

(answer: $\mu_{\text{LOWER}} = 2.20$ and $\mu_{\text{UPPER}} = 13.06$)

The exact limits on μ , for any specified count c and confidence coefficient $(1-2\alpha)$, can -- without trial and error -- be found directly from the χ^2 distribution.

$$\mu_{\text{LOWER}} = (1/2) \chi^2_{\alpha, df = 2c}$$

$$\mu_{\text{UPPER}} = (1/2) \chi^2_{1-\alpha, df = 2(c+1)}$$

Values of χ^2 for any α and df are readily available from many statistical packages or spreadsheets, or can be found from an adequately extensive tabulation of the χ^2 distribution.

In our example... $c = 6$; $(1-2\alpha) = 0.95$,

so $\alpha = 0.025$, $1-\alpha = 0.975$.

$$\mu_{\text{LOWER}} = (1/2) \chi^2_{0.025, 12df} = (1/2)4.40 = 2.20,$$

$$\mu_{\text{UPPER}} = (1/2) \chi^2_{0.975, 14df} = (1/2)26.12 = 13.06.$$

If you use Excel, reverse α and $1-\alpha$.

Clever way to obtain exact limits, using Stata

use 2-sample comparison, with 'infinite' comparison group:-

`epitab` syntax is `iri #a #b #N1 #N2 [, level(#)]`

so set #b and N2 to be very large (a (our c)=6 events in 1 person-year, versus b=1000000 events in 1000000 person years:

```

c very_large# PT very_large_PT
iri 6 1000000 1 1000000

```