



A new look at Halley's life table

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Summary. Edmond Halley published his Breslau life table in 1693, which was arguably the first in the world based on population data. By putting Halley's work into the scientific context of his day and through simple plots and calculations, new insights into Halley's work are made. In particular, Halley tended to round his numbers and to massage his data for easier presentation and calculation. Rather than highlighting outliers as would be done in a modern analysis, Halley instead smoothed them out. Halley's method of life table construction for early ages is examined. His lifetime distribution at higher ages, which is missing in his paper, is reconstructed and a reason is suggested for why Halley neglected to include these ages in his table.

Keywords: Data analysis; Demographic statistics; Life tables; 17th century; Smoothing

1. Introduction

In 1693 Edmond Halley constructed a life table or more correctly, as noted by Greenwood (1941, 1943), a population table. It was based on data collected for the years 1687–1691 from the city of Breslau, which is now called Wrocław, by the Protestant pastor of the town, Caspar Neumann. The data that Halley used were the numbers of births and deaths recorded in the parish registers of the town, which was then under the control of the Habsburg monarchy of Austria. It was a town in a Polish area comprised predominantly of German speakers adhering to Lutheranism within an officially Roman Catholic empire (Davies and Moorhouse (2002), pages 159–160, 180). Halley published the table in the Royal Society's *Philosophical Transactions* (Halley, 1693). Over the years there have been many who have written about this life table, its origin and possible mode of construction. For example, during the 1840s when the insurance industry was rapidly expanding in Britain, the actuary Edwin Farren wrote a book on the development of life insurance that contains a *précis* of Halley's paper. Farren, who was also a member of the Statistics Section of the British Association for the Advancement of Science, commented that Halley's life table (Farren (1844), page 26)

'seems not only to the general reader as the first complete "Table of Mortality" upon record, but also as constituting the first real step in the *art* of Life-measurement'.

During the 20th century, statisticians continued to look at Halley's life table. Pearson (1978), pages 74–81, described some of the 17th-century source material that can be used to reconstruct how Halley came to work on his life table. Using analyses of Halley's work from the 19th century, Hald (1990), pages 131–141, described the background to the table and its possible method of construction.

The raw data, which were obtained from Neumann, from which Halley constructed his table,

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Table 1. Halley's Breslau table

<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>
1	1000	8	680	15	628	22	586	29	539	36	481	7	5547
2	855	9	670	16	622	23	579	30	531	37	472	14	4584
3	798	10	661	17	616	24	573	31	523	38	463	21	4270
4	760	11	653	18	610	25	567	32	515	39	454	28	3964
5	732	12	646	19	604	26	560	33	507	40	445	35	3604
6	710	13	640	20	598	27	553	34	499	41	436	42	3178
7	692	14	634	21	592	28	546	35	490	42	427	49	2709
<hr/>													
<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	<i>Age. Curt.</i>	<i>Persons</i>	56	2194
												63	1694
<hr/>													
43	417	50	346	57	272	64	202	71	131	78	58	70	1204
44	407	51	335	58	262	65	192	72	120	79	49	77	692
45	397	52	324	59	252	66	182	73	109	80	41	84	253
46	387	53	313	60	242	67	172	74	98	81	34	100	107
47	377	54	302	61	232	68	162	75	88	82	28		34000
48	367	55	292	62	222	69	152	76	78	83	23	Sum	Total
49	357	56	282	63	212	70	142	77	68	84	20		

have not survived. They were reconstructed in the 1880s by Jonas Graetzer, the medical officer in Breslau at that time. In Graetzer's day, Breslau was part of the German Empire. The data that he collected are published in Graetzer (1883). Of importance to subsequent studies of Halley's work, Graetzer (1883) provided the number of deaths at each year of age from 1 to 100 years for each of the five years 1687–1691. Also given are the numbers of births in each of the five years. The partial data that Halley published are, in most cases, close to what Graetzer extracted; however, there is not exact agreement. On the 200th anniversary of Halley's analysis, Richard Böckh, who was director of the Statistical Office of the City of Berlin and a friend of Graetzer (Anonymous, 1948), constructed a table that was similar to Halley's by using Graetzer's data and the modern demographical tools of his time (Böckh, 1893). Both Graetzer's and Böckh's work are in German. It is this work that Hald has made available to an English readership.

The data that are analysed in the paper can be obtained from

<http://www.blackwellpublishing.com/rss>

Halley's life table is shown in Table 1. The body of the table shows the number of lives at various ages, specified as 'Age Curt.' or 'age current'. The appellation '1 age current', for example, means that the individual is in his first year of life so the first birthday has yet to be reached. Ages 1–84 years appear along with the number alive at each of those ages, all arranged in groups of 7. A little arithmetic on the numbers within the table verifies that the two rightmost columns of Table 1 show the number of people alive in each of the 13 age groups 1–7, 8–14, . . . , 78–84 and 85 years and older. The numbers of people alive for each of the ages over 84 years do not appear in Table 1 but the sum of these numbers for the 16 years is given as 107. Strictly speaking, Halley's table is a population table, not a life table. For the city of Breslau at the beginning of the 1690s with a total population of about 34000 people, the table shows estimates of the number in the population at each integral age up to age 84 years. It can be used as a life table to find probabilities of death and survival at various ages.

Given that Halley's work has been scrutinized from a historical perspective for more than 165

years, one might ask whether there is anything new to be found or said. Using some standard actuarial, statistical and historical tools, I believe that there is. In Section 2, on the basis of an experiment that Halley carried out, I infer generally how Halley approached estimation and data analysis. I also examine the original intention behind Neumann's data collection and how this possibly had an influence on subsequent analysis. Section 3 is devoted to the question of why Halley smoothed the mortality data that he obtained from Neumann. Section 4 examines Halley's results for the early ages in his table and speculates on why these results differ from Neumann's data. Section 5 deals with the end of the table for which Halley provides no numbers other than that the number of individuals living between ages 85 and 100 years inclusively is 107.

2. How Halley dealt with data

The Breslau mortality data came to Halley in a roundabout way. The route appears to be from Caspar Neumann to Gottfried von Leibniz to Henri Justel to the Royal Society to Edmond Halley. As intermediaries, Leibniz worked as court librarian to the Elector of Hanover and Justel was the Royal Librarian in England. To piece the route together, see Harnack (1900), page 137, Leibniz (1964), page 605, Leibniz (1970), pages 275–279, and Pearson (1978), pages 75–76.

Originally, Neumann wrote to Leibniz in Hanover about his research into population statistics. Specifically Neumann was looking for patterns in births and deaths in Breslau that would relate to his interests in physicotheology, the study of the evidence for divine design in the natural world. On May 24th, 1692, Leibniz wrote Justel a long letter with much scientific news (Leibniz, 1970). In the last paragraph Leibniz wrote that Neumann had made some good points about the births and deaths in Breslau. In particular, Leibniz noted that there was no substance to the notion of climacteric years. Leibniz probably sent Neumann's work to Justel at about that time. The idea of climacteric years dates from antiquity. They were considered the critical years of human life and so more deaths were expected in those years. Ancient researchers had a variety of opinions on what these years were. Every seventh year was one possibility and every ninth year was another. It was thought that the 63rd year was a particularly critical year and therefore was called the grand climacteric. Others thought that the grand climacteric was either the 49th or the 81st year. Climacteric years are defined and discussed in the *Oxford English Dictionary*. After Justel received the data from Leibniz, he presented it to the Royal Society on January 18th, 1693, at which time the members perused Neumann's data. The Royal Society's *Journal Book* records that

‘... the Climactericks are rigorously examined, to see if there be any thing worth remark which by these notes seem to argue to be a Groundless Conceit’.

Like Leibniz, the main focus was on dismissing the validity of climacterics. Neumann's purpose in presenting the data appears to have been to dispel any superstitions about these years, as well as other superstitions such as the possible connection between the number of deaths and the phases of the moon. Halley became involved with the data in his position as clerk to the Royal Society. He appears to have taken the data and worked on his analysis between February 8th and March 8th, 1693, at which point he presented his mortality table to a meeting of the Royal Society. A week later he presented some annuity calculations based on his table (see volume 8 of the Royal Society's *Journal Book*).

Two of Halley's applications of his life table were motivated by the military campaigns between 1689 and 1697 of King William III and his allies against France, known as the Nine Years' War. In his list of five applications, the first to appear on Halley's list is that his table can be used

to find the proportion of the population able to bear arms, which is the proportion of men between the ages of 18 and 56 years. An earlier attempt at answering this kind of question was made by John Graunt in his work on the London bills of mortality. See, for example, Graunt (1676), pages 83–85, which is the edition with which Halley was most likely acquainted. The last item on Halley's list, which is almost an afterthought like his second presentation to the Royal Society, is the valuation of life annuities, which we shall delve into further in Section 5. Through what is now known as the Million Act of 1692, the government intended to raise £1 million to finance William's war (Walford (1871), page 104). The money was to be raised by the sale of life annuities. This kind of offering to the general public, which was an idea imported from Holland, was a financial innovation that was entirely new to England (de Vries and van der Woude, 1995). The annuities were secured by an excise tax on beer and liquor rather than land. On the basis of his valuations, Halley commented that for many it was advantageous to buy these annuities. They were priced at seven times the annual payment regardless of age whereas a proper price, according to Halley's tables, was more than seven times the annual payment until age 65 years.

An examination of how Halley dealt with data in other contexts might provide some insight into his mortality table construction. Consequently, I examined an experiment that was carried out by Halley on water evaporation a few years before his work on the Breslau mortality data. The results appear in Halley (1687). Beyond the actual experiment, the paper is informative about Halley's practice of rounding numbers and making simplifying assumptions.

Halley carried out his experiment for the purpose of estimating the quantity of water that would evaporate from the sea after noting that water flows into the sea from rivers and yet the sea level remains constant. He took a pan of water, heated it to the hottest outdoor temperature experienced on a summer day and then found the weight of water that evaporated over a two-hour period. He made various calculations, based on the size of the pan and the known weight of a cubic foot of water, to translate the weight that he obtained into the thickness of the water that evaporated from the pan. All his calculations were done by hand so at times he rounded numbers to convenient fractions for ease of calculation. For example, when he found that the thickness of the water evaporated from the pan was $35/1862$ of an inch, he changed it to $1/53$, rather than $1/53.2$, the exact value. He then went on to say 'but we will suppose it only the sixtieth part, for facility of Calculation'. This conveniently comes to a tenth of an inch as a lower bound for surface evaporation over a 12-hour period. Later in the paper Halley tried to estimate river flows. He considered a bridge over the Thames above the effect of the tide water and did a simple calculation. At this location he estimated the width of the river to be 100 yards and the depth 3 yards. With a water speed estimated at 2 miles per hour, he found that over 24 hours a total of 25344000 cubic yards of water would flow into the sea. What can be seen in this paper is Halley rounding numbers for convenience and making simplifying assumptions when necessary to obtain his results.

Knowing that Halley tended to make simplifying assumptions and to round his numbers for convenience, an interpretation of Halley's analysis of Neumann's data may proceed, perhaps with slightly improved insight.

3. Halley's initial smoothing of the data

Table 2 shows Halley's list of deaths in Breslau by age as he published them (Halley (1693), page 599). The entries in the table are the average number of deaths at each age for the 5 years 1687–1691. Where the ages are abbreviated to intervals, Halley indicated that the corresponding number of deaths is the average over the 5 years and the ages in the interval. For example, for the

Table 2. The data as Halley presented them

Age Curt.	Deaths	Age Curt.	Deaths	Age Curt.	Deaths	Age Curt.	Deaths	Age Curt.	Deaths
7	11	22–26	6.5	45	7	64–69	9.5	84	2
8	11	27	9	46–48	7	70	14	85–89	1
9	6	28	8	49	10	71	9	90	1
10–13	5.5	29–34	7	50–53	?	72	11	91	1
14	2	35	7	54	11	73–76	9.5	92–97	?
15–17	3.5	36	8	55	9	77	6	98	0
18	5	37–41	9.5	56	9	78–80	7	99	0.2
19–20	6	42	8	57–62	10	81	3	100	0.6
21	4.5	43–44	9	63	12	82–83	4		

age interval 64–69 years, the entry 9.5 is the average number of deaths for the ages 64, 65, . . . , 69 years over the five years 1687–1691. Never explicitly stated, Halley gives the impression that all he has done by constructing the intervals is to save print space while keeping the original data intact.

If we look at the complete data as collected by Jonas Graetzer, we can see why Halley grouped some of his ages. A plot of the total number of deaths over the five years 1687–1691 taken from Graetzer (1883) is shown in Fig. 1. I have plotted only ages 5 years and upwards. Including in Fig. 1 the high infant mortality that is typical of the period would distort the graph by visually dampening the variation in the deaths at the higher ages. Also, Halley treated the younger ages, 6 years and below, separately in his analysis. What immediately stands out in Fig. 1 is that at a number of ages divisible by 5, particularly between ages 40 and 80, the number of deaths is high compared with nearby years. This is a reflection of some surviving relatives not remembering their kin's exact age and reporting a rounded or approximate number to the church clerk. Halley may have recognized this problem but did not mention it in his paper. Of the ages that have a



Fig. 1. Total number of deaths at Breslau, 1687–1691: ●, quinquennial years; ○, other years

particularly high number of deaths (40, 50, 55, 60, 65, 70 and 75 years), four have been placed in a group and their unusual values have been hidden by averaging or by dropping the data altogether as in the case of age 50 years. I conjecture that Halley did this intentionally; he was concealing evidence of possible climacterics at 5-year intervals, the general concept of which Royal Society members had called a ‘groundless conceit’ at an earlier meeting.

4. Early ages in the table

For the youngest age group, Halley stated that, over the five years 1687–1691, the average number of births is 1238. He then says,

‘I will suppose the People of Breslaw to be increased by 1238 Births annually. Of these it appears by the same Tables, that 348 do die yearly in their first Year of their Age, and that but 890 do arrive at a full Years Age; and likewise, that 198 do die in the Five Years between 1 and 6 compleat, taken at a Medium; so that but 692 of the Persons born do survive Six whole Years.’

Halley’s and Graetzer’s numbers do not agree for this age group. Graetzer says that the average number of deaths per year in the first year of life is 353, compared with Halley’s 348. Graetzer found that the average number of deaths over ages 2–6 years is 189.2; Halley says it is 198. Despite these discrepancies and taking into account Halley’s knack for a slight massaging of the data, Halley’s life table can be derived from the existing information. This is done by using Böckh’s approach to life table construction and an appropriate selection of the data.

Start with the 1238 births and 348 deaths (Halley’s number) in the first year of life. The average number alive at age current 1 year is then the mean of the two numbers at the beginning and the end of the year, or $\{1238 + (1238 - 348)\}/2 = 1064$. From the number alive at the end of the first year, subtract the deaths at each subsequent year using Graetzer’s data. The number of lives at each age current is the average of the number of lives at the beginning and the end of the year. As discussed in Hald (1990) and originally in Böckh (1893), this is the same as calculating, in standard actuarial notation, $L_x \cong (l_x + l_{x+1})/2$, where l_x is the number of people who are alive at age x and L_x is the average number living between ages x and $x + 1$. I have rounded my numerical values in these calculations to the nearest integer and set them out in Table 3. When Halley’s values from Table 1 are put beside these calculations, there are some differences. However, the sum of the differences is small (–1). From his earlier experiment on the evaporation of water, Halley showed his inclination to round data and to simplify. Therefore, it is possible that Halley rounded the 1064 in his calculation to an even 1000 and then distributed his 64 lives into some

Table 3. Calculations at the beginning of the table

<i>Age (years)</i>	<i>Lives</i>	<i>Average number of deaths</i>	<i>Age Curt.</i>	<i>Lives</i>	<i>Halley's values</i>	<i>Difference</i>
0	1238	348.0	1	1064	1000	64
1	890	93.6	2	843	855	–12
2	796.4	44.6	3	774	798	–24
3	751.8	23.6	4	740	760	–20
4	728.2	12.2	5	722	732	–10
5	716	15.2	6	708	710	–2
6	700.8	11.6	7	695	692	3
7	689.2		Sums	5546	5547	–1

of the early ages of the table, thus slightly fudging the data. By redistributing the excess lives over the next 6 years, Halley maintained approximately the same population size in the earliest age group, 5547.

Like Halley's number 1000, modern life tables begin with convenient numbers such as 1000 or higher in multiples of 10. The number in modern tables is called the radix of the table and in actuarial notation is given by l_0 . The radix is an arbitrarily convenient starting point in a life table for the number of lives at age 0. This does not apply to population tables. As Greenwood (1941) has stressed, Halley's number is not the radix of the table; nor is it just a convenient number. Rather, it has meaning for a specific population. It is the approximate number of people who were alive in Breslau among those living who were yet to reach their first birthdays. In actuarial notation, Halley's 1000 is L_0 for Breslau.

This being said, the interpretation of the numbers in Halley's table given here differs slightly from that given by Greenwood (1941). After rightly commenting that Halley did not assume that the people of Breslau formed a stationary population since the average number of births *per annum* was greater than the average number of deaths, Greenwood went on to give his own interpretation of 1000 for L_0 . According to Greenwood, given 1238 births and 348 deaths in the first year of age, and $L_0 = 1000$, that must mean that there were 238 deaths in the first half of the year and 110 in the second half. This is equivalent to saying that 68% of the deaths in the first year occur in the first half of the year, which is an amount that Greenwood noted was close to 73.5%, a percentage from some English life tables in the first decade of the 20th century. Given the evidence for Halley's penchant for rounding and modest massaging of the data, I believe that Greenwood anachronistically has injected too much sophistication into Halley's analysis.

5. The end of the life table

For age current 85 years and upwards, unlike the rest of his life table, Halley did not provide the number of lives at each age. Rather, in the margin of his table as shown in Table 1, Halley specified that the total number of people from ages 85 to 100 years is 107. Others who knew Halley personally have tried to fill in the end of the table within 40 years of its original publication and within Halley's lifetime. Attempts were made by the mathematicians Brook Taylor and William Jones, who were both Fellows of the Royal Society as was Halley. It is tempting to speculate, and I shall, why Halley did not give the numbers to Taylor or to Jones directly, but let them guess for themselves. Taylor's numbers appear in a manuscript in St John's College Library, Cambridge (catalogued as TaylorB/C3), which may be dated to *circa* 1725–1730; and Jones's numbers appear in a manuscript in the Macclesfield collection at Cambridge University Library (catalogued Add. 9597/4/38). Jones's manuscript may be dated to *circa* 1733–1734. Each of Taylor's and Jones's extensions to Halley's table is shown in Fig. 2. In both cases there is a fairly smooth transition from Halley's original to what the other two have predicted. Of interest is that Taylor's tabular lifespan is only 90 years whereas Jones's is 100 years, which is the full extent given by Halley's table. Further, Taylor's number of lives between ages 85 and 100 years sums to 51 whereas Jones's number of lives sums to 105.099. In neither case is the sum 107; however, Jones appears to provide a better extension. Neither Taylor nor Jones has provided any justification for their respective extensions, only the numbers.

Both Jones and Taylor use Halley's life table, with augmentations, to evaluate the present value of a life annuity of £1 *per annum* at an annual interest rate of 5%. When Halley made his annuity calculations, the legal rate of interest was 6%; the legal rate was reduced to 5% in 1714 (Walford (1871), page 113). Generally, the value of a life annuity is the expectation of future payments, taking into account interest, where the expectation is taken with respect to

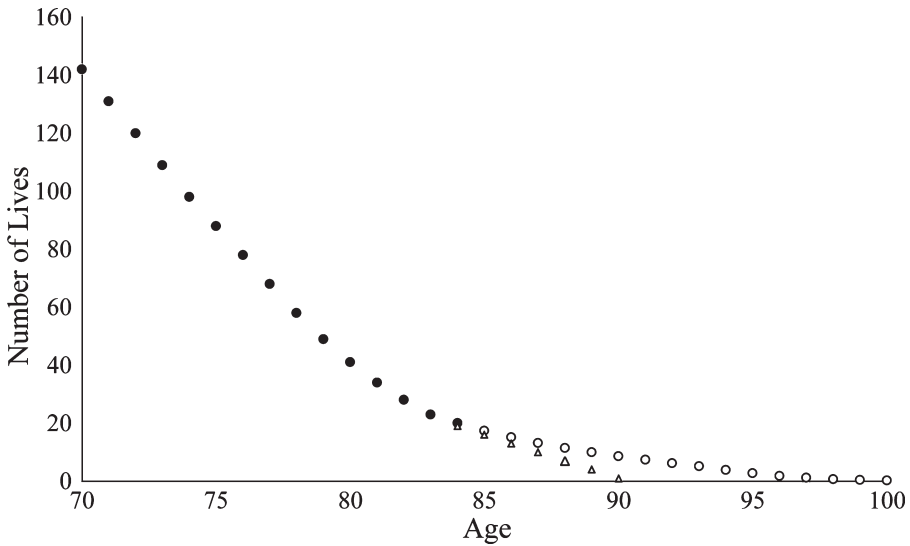


Fig. 2. Jones's (○) and Taylor's (△) extension to Halley's (●) life table

Table 4. Life annuity of £1 paid at the end of every year based on 6% interest

Age at issue (years) (1)	Values taken from Halley's table (2)	Value of annuity issued at age 1 year (3)	Corrections to Halley's annuity values (4)	Annuity values based on 107 lives (5)	Annuity values based on 155 lives (6)
1	10.28	—	10.28	10.28	10.28
5	13.40	10.504	13.01	13.01	13.01
10	13.44	10.275	13.44	13.44	13.44
15	13.33	10.318	13.19	13.18	13.18
20	12.78	10.279	12.78	12.77	12.77
25	12.27	10.279	12.27	12.26	12.26
30	11.72	10.278	11.72	11.71	11.71
35	11.12	10.275	11.12	11.14	11.15
40	10.57	10.278	10.57	10.54	10.56
45	9.91	10.278	9.91	9.88	9.88
50	9.21	10.278	9.21	9.17	9.18
55	8.51	10.277	8.51	8.47	8.49
60	7.60	10.277	7.60	7.55	7.58
65	6.54	10.277	6.54	6.48	6.52
70	5.32	10.277	5.32	5.24	5.32

the survival distribution given by the life table. In standard actuarial notation, the value of the life annuity is denoted by a_x when issued at age x . Jones and Taylor must have interacted in some way concerning Halley's life table. Jones copied out a list of values, which were attributed by Jones to Taylor, of the values of a life annuity for issue ages 1–72 years with interest at 5% (Add. 9597/4/38). Curiously, this differs from the manuscript containing Taylor's valuations of the life annuity at 5% (TaylorB/C3).

In view of the missing data at the end of Halley's life table, there is no way to carry out any accurate annuity valuations based on the published table. Yet, later in his paper, Halley

provided a table showing the present value of a life annuity of £1 at 6% interest, in particular a_x for $x = 1, 5, 10, 15, \dots, 65, 70$ years. The values are shown in column (2) of Table 4. These can be used to reconstruct the end of Halley's table.

First, it is necessary to check the correctness of Halley's annuity valuations. This can be done in the following way. Assume that one of the annuity values is correct—say $a_1 = 10.28$, the annuity value at age 1 year. Then, using the relationship

$$a_x = \frac{1}{l_x} (vl_{x+1} + v^2l_{x+2} + \dots + v^n l_{x+n} + v^n l_{x+n} a_{x+n}) \tag{1}$$

and Halley's life table, it can be verified whether or not the remaining values are correct. In equation (1), $v = (1 + i)^{-1}$ where i is the rate of interest. For $x = 1$ and $n = 4, 9, 14, \dots, 69$, the value of a_1 was recalculated from the values of $a_5, a_{10}, a_{15}, \dots, a_{70}$ given in column (2) of Table 4. The recalculations are shown in column (3) of Table 4. With the exception of two ages, 5 and 15 years, the value of a_1 is correctly reproduced to two decimal places. It may be safely assumed that Halley has calculated a_5 and a_{15} incorrectly. Assuming that a_5 and a_{15} are the only two incorrect values, it may be further assumed that Halley did not use a recursive relationship such as equation (1) to calculate a_x . If he had done that, then a_1 and a_{10} would also be incorrect. Since Halley was making annuity valuations by hand using tables of logarithms and since the number of calculations increases as the ages become younger, his errors are not completely unexpected. Assuming that $a_1 = 10.28$ is correct, then equation (1), along with Halley's life table, can be used to make corrections to a_5 and a_{15} . These are shown in italics in column (4) of Table 4.

Now that we have the correct values for the annuities based on Halley's life table, we can try various scenarios of the distribution of lives after age 84 years to see which one might conform to the annuity calculations given in column (4) of Table 4. In that way we can reconstruct the rest of the table. The first question to ask is whether the number 107 is the correct number of lives beyond age 84 years. The easiest way to check this is to look at the extreme case: the one that gives the highest value for the annuity at each of the issue ages in Table 4. The extreme case is when there are no deaths in the 85th–89th years, 13 deaths in the 90th year and seven deaths in the 91st. In that case there are 20 people alive in each of the five ages 85–89 years and seven alive at 90 years of age for a total of 107 lives, which agrees with Halley's total. The annuity calculations are shown in column (5) of Table 4. As the age of issue increases, the annuity is increasingly undervalued, which reflects an inadequate number of lives at the end of the table. Consequently, I tried several scenarios, by trial and error, to try to reproduce Halley's annuity values. Among several possible distributions of lives at the end of the table, that which best fits Halley's annuity calculations has 155 lives at ages 85 years and over. The annuity values for this distribution are shown in column (6) of Table 4. It is obvious that Halley did some slight fudging of the presentation

Table 5. Conjectured end of Halley's life table

Age Curt.	Persons	Age Curt.	Persons	Age Curt.	Persons
85	19	90	13	95	6
86	18	91	12	96	5
87	16	92	11	97	4
88	15	93	9	98	3
89	14	94	7	99	2
				100	1

of his data so that the population of Breslau amounted to 34000 souls exactly. Since he carried out this data massaging, he was unwilling to give the correct values to William Jones, Brook Taylor or anyone else. Alternatively, he may have lost the original data since Taylor's and Jones's tables were drawn up about 30–40 years after Halley first worked on the data. Table 5 shows the conjectured distribution of lives at the end of the life table based on the 155 lives.

6. Conclusion

By modern standards there are flaws in Halley's data analysis and he did fudge and massage his data. At the same time, it is important to consider the historical context, particularly the difficulty and tedium of arithmetical calculations done by hand and the fact that Halley's work was testing uncharted waters. Despite the flaws that I have uncovered, Halley's work remains a brilliant contribution to demographic statistics, data analysis and other areas that involve life table construction.

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