Course 607 (Inferential Statistics)
Supplementary Examination
Nov 19, 1993

Name: ______________________

____

1. Scores (in a reference population) on the college board SAT Verbal exam are normally distributed with mean 500 and standard deviation 100.
   What percent of scores are above 700?

2. Scores (in a reference population) on the college board SAT Verbal exam are normally distributed with mean 500 and standard deviation 100.
   What scores make up the top 25% of this distribution?

3. You work for a large plant nursery that sells transplants of shrubs by mail. One species is sold when 12" tall. You want to advertise "Fast growth! Our 12-inch transplants grow to _______ inches in only five years!" You know that actual growth over five years varies among individual shrubs according to a normal distribution with mean 30" and standard deviation 5". Fill in the blank in the advertisement so that 75% of the shrubs will grow at least as much as claimed.

4. A study of the career path of college faculty who eventually reach the rank of Professor looks at the time (in years) X spent in post-doctoral positions before appointment as Assistant Professor, the time Y until promotion to Associate Professor, and the additional time Z until promotion to Professor. The study finds that Y and Z are not independent because both are small in the case of the most successful faculty members. The study also finds the means and standard deviations of these random variables to be as follows:

   $\mu X = 1.3$ SD(X) = 2.3
   $\mu Y = 5.2$ SD(Y) = 1.4
   $\mu Z = 6.9$ SD(Z)= 4.3

   Can you find the mean of the total time X + Y + Z from the doctorate degree until promotion to Professor from this information? If so, do it. If not, say why not.

5. Can you find the standard deviation of the total time X + Y + Z from this information? If so, do it. If not, say why not.

6. An African hospital is treating patients who have Lhassa Fever. Seventy percent of patients with this disease die, and each patient lives or dies independently of other patients. Over a long period, the hospital expects to treat 150 patients. What is the probability that at least 100 of these will die?

7. In a large population of college students, 20% of the students have experienced feelings of math anxiety. If you take a random sample of 10 students from this population, the probability that exactly 2 students have experienced math anxiety is
(a) .3020
(b) .2634
(c) .2013
(d) .5
(e) 1

8. If a population has a standard deviation σ, then the standard deviation of mean of 100 randomly selected items from this population is
(a) σ
(b) 100σ
(c) σ/10
(d) σ/100
(e) .1

9. An opinion poll asks a random sample of voters, "Do you think elected government officials are underpaid?" Suppose 25% of the population would respond "yes." If the sample size is 400, the probability that at least 90 respond "yes" is approximately
(a) .875
(b) .125
(c) .750
(d) .225
(e) none of the above

10. Cholesterol level in a particular male population is assumed to follow a normal distribution with standard deviation 21 units. You want to estimate the population mean cholesterol level within ±10 with 95% confidence. How large a sample must you take?

11. Packages of frozen peas are supposed to have a mean weight of 12 oz. The manufacturer wishes to detect either too low a mean weight (which is illegal) or too high a weight (which reduces profit). Experience shows that the weights have a normal distribution and that the standard deviation of the population of weights is σ = 0.6 oz. even when the mean changes. State H0 and Ha for the manufacturer's problem.

12. You want to test whether or not the mean weight is 12 oz. If a sample of 25 packages gives a sample mean weight of xbar = 12.3 oz., is the result significant at the 10% level? at the 5% level?

13. Scores on a law school aptitude test are normally distributed with standard deviation σ = 100 points. The national mean is 500 points, but some groups of students may have different means. A psychologist conjectures that students from low income families score below the national average. She tests a sample of 36 students from homes with annual incomes below $10,000. The mean score in this sample is

xbar = 462 points. What are H0 and Ha for this test?

14. The concentration of active ingredient in a pharmaceutical product is very important, since either too much or too little can affect the drug's performance. A particular product is supposed to contain 10% of the active ingredient. During production, 3 specimens from each batch are taken for assay. The last sample has mean concentration xbar = 10.4%. You can assume based on past data that the concentration for this production process varies according to a normal distribution with standard deviation $\sigma = 0.2\%$. Calculate an appropriate test statistic and report the P-value to see if the assay data give evidence that the mean concentration of the batch is not 10%.

16. Systolic blood pressure in elementary school children varies from child to child according to a normal distribution with standard deviation about 10. You want to estimate the mean systolic blood pressure among the children of Basque sheepherders to within ±3 with 90% confidence. How many children must you observe?

17. Three specimens (A,B,C) were weighed prior to a chemical reaction, with results

A	B	C
8.26 g.	8.27 g.	8.25 g.

The same specimens were subjected to the chemical reaction, with possible change in weight. They were weighed again to obtain

A	B	C
8.15 g.	8.14 g.	8.12 g.

Is there significant evidence at 5% level that the reaction decreases the mean weight of specimens such as these? (Be sure to state the hypotheses.)

18. A government agency is concerned about deterioration of automobile anti-pollution arrangements after the cars are sold. The agency took a sample of 19 new cars at dealers, and a year later took a second sample of 28 cars which had been in service for at least 8 months. The NOX (nitrogen oxides) readings were as follows:

	Sample Size	Sample Mean	Sample Stnd. Dev.
New	19	53.3 ppm	3.3 ppm
Used	28	56.8 ppm	4.6 ppm

Is there strong evidence that the means are different? (State the hypotheses, carry out the test, use a table to give bounds between which the P-value falls, and state your conclusion.)

19. It's hard to lose weight. People spend a lot of money on weight-loss programs. Can a simple and inexpensive program do just as well as an elaborate and expensive program? You want to compare

the following two weight-loss programs:

Program A: (Minimal program) Subjects meet with a counselor who offers advice on how to lose weight gradually through exercise and a sensible diet. A second meeting with the counselor is scheduled 10 weeks later to discuss the subject's progress in losing weight.

Program B: (Standard program) Subjects meet with a counselor once a week for 10 weeks. They are weighed each week, and the counselor offers encouragement and more elaborate advice on weight-control strategies.

In an experiment designed to compare the effectiveness of the two programs, subjects were randomized to the two programs. There were 17 subjects in Program A and 16 in Program B. Their weight loss from the start of the program was measured six months after the program had ended. Here are the summary statistics:

Group	n	Mean	Std Dev
Program A	17	5.52 pounds	9.58 pounds
Program B	16	11.05 pounds	9.83 pounds

It appears that the standard program produces a larger average weight loss. But perhaps the difference is just due to chance. Is there evidence that the average mean weight loss of overweight people would be greater for the standard program than for the minimal program? To answer this question, state hypotheses, do a test, and obtain a P-value (use tables to find numbers that P lies between). Then state your conclusion.

20. A nursery advertises that 90% of its stock survives transplanting. An apartment complex purchases 80 trees from this nursery, and 11 do not survive. Give a 95% confidence interval for the proportion of trees that survive.

21. The Helsinki Heart Study was a randomized comparative experiment that asked whether taking the drug gemfibrozil (which lowers cholesterol levels) would prevent heart attacks in middle-aged men with high blood cholesterol. In this study, 2051 men took gemfibrozil; 56 of them had heart attacks during the five years of the study. Another 2030 men took a placebo, and 84 of these men had heart attacks. The study gave highly significant evidence that taking gemfibrozil does reduce the incidence of heart attacks. (Don't do the test of significance.) So we want to know how much the drug reduces the proportion of men like those in the study who will have a heart attack within five years. Give a 99% confidence interval to answer this question.

22. An historian examining British colonial records for the Gold Coast in Africa suspects that the death rate was higher among African miners than among European miners. In the year 1936, there were 223 deaths among 33,809 African miners and 7 deaths among 1541 European miners in the Gold Coast. (Data courtesy of Raymond Dumett,

Department of History, Purdue University.) Consider this year as a sample from the pre-war era in Africa. Is there good evidence that the proportion of African miners who died during a year was higher than the proportion of European miners who died? [State hypotheses, calculate a test statistic, give a P-value as exact as the tables in the text allow, and state your conclusion in words.]

Course 607 (inferential statistics)
Answers for supplementary examination of Nov 19, 1993

1. z = (700 - 500) / 100 = 2; P(Z > z) = 2.28%.
2. 25% above Z=0.67 i.e. 25% above 500 + 100*0.67 i.e. above 567.
3. 75% above Z= –0.67; i.e. 75% above 30" - 0.67*5" i.e. above 26.65"
4. μ(Sum) = sum(μ) =13.4 years (whether correlated or not)
5. No. Variances do not add unless the random variables are uncorrelated.
6. Binomial with n=150 and π=0.7; Need Prob (≥100 | π=0.7 and n=150)
   Too large for Binomial tables so use Gaussian approxn. to Binomial
   Say we work with count; Need μ (or E) and SD. E = μ = nπ = 105 and
   SD = √ n(π[1–π] = 5.6.
   Prob(100 or more) = Prob (99.5 or more on continuous scale)
   z = [99.5 - 105] / 5.6 = –0.98; Prob (Z > z) = 0.8133;
   If work with proportions, everything scaled down by 100 but z calculation the same.
7. Binonial with n=10 and π=0.20. See Binomial table or Moore &McCabe for (a) .302
8. SEM = σ /√n ; so (c)
9. Binomial with n=400 and π=0.25; Need Prob (≥90 | π=0.25 and n=400)
   Too large for Binomial tables so use Gaussian approxn. to Binomial
   Say we work with count; Need μ (or E) and SD. E = μ = nπ = 100 and
   SD = √ n(π[1–π] = 8.66.
   Prob(90 or more) = Prob (89.5 or more on continuous scale)
   z = [89.5 - 100] / 8.66= –1.21; Prob (Z > z) = 0.89 (a);
10. Margin of error (i.e. ±) =10= z 21/√n =>n = z^2 * 21^2 / 10^2 = 1.96^2 * 21^2 / 100 =17
11. H_0: μ = 12, H_a: $\mu \neq 12$.
12. SEM=0.6/√25 = 0.12 so 12.3 is 0.3 / 0.12 = 2.5 SEM's above μ_0=12.The portion
    of the Gaussian (Z) distribution beyond ±2.5 is only 0.012 or 1.2%. So yes, yes.
13. H_0: μ = 500, H_a: μ < 500
14. 2-sided issue; 10.4 is [10.4–10] / (0.2/√3) = 3.46; Prob(|Z| >z = 3.46 is P = .00054.
16. Margin of error =3= z 10/√n =>n = z^2 * 10^2 / 3^2 = 1.645^2 * 10^2 / 9 = 30.x so 31.
17. H_0: μ = 0, H_a: μ < 0, where μ represents the mean loss in weight.
    t = 12.3 / (1.15/√3) = 18.5. the critical value for t_2 = .05 (1-sided) is 2.92. P<0.05.
18. H_0: $\mu_1 = \mu_2$, H_a: $\mu_1 \neq \mu_2$. t for 2 indep samples = 3.036. df=45 0.005 < P < 0.010.
    There is strong evidence to suggest a difference in the μ's.
19. H_0: μ_A=μ_B, Ha: $\mu_A \neq \mu_B$, t = -1.54, df = 31, 0.10 < P < 0.20. This experiment does not provide strong evidence to distinguish between the effectiveness of the two programs.
20. proportion = 69/80 = 0.86;
    95% CI: p ± 1.96SE(0.86) = 0.863±1.96*$\sqrt{0.86*0.14 / 80}$
21. 0.0141 ± 2.256*SE(difference in proportions) = 0.0141 ± 2.256(0.0057) or ± 0.0146
22. H_0: $\pi_1 = \pi_2$, H_a: $\pi_1 \neq \pi_2$, where $\pi_1$is the death rate for the Africans and π_2 is the death rate for the Europeans. X^2 = 0.961, P > .25. (Alternatively, z = .98, P = .327.) There is no evidence to conclude that the death rates are different. Note that the problem statement suggests that a one-sided alternative is reasonable. In this case, the P value for the z statistic would be halved (.163). The X^2 statistic is not appropriate for the one-side alternative.

Plasma Caffeine

1 Convenience sample. Might have looked for plasma in community samples gathered for other purposes, as in a health survey etc. Would want what time of day samples taken etc.

2 Because of variation in reagents etc., some variation if samples containing a known quantity is assayed by different kits (batches). SD(this variation) is 5.5% of mean quantity.

3 Distribution is very skewed, so %iles would be better.

4 Have no problem with t-test (large n, so skewness of individual values not an issue). If really fussy, could use nonparametric analog (rank sum test).

5 Maybe they had to have two cups of strong coffee just to face the stress of the biochemical investigation. Can one generalize to 'in general' from one (non-random) day?

6 Depends on whether interested in acute effects or chronic ones. If the latter, would need more than one sample per individual to get a good idea of the longterm levels.

Early discharge of preterm infants

1 There have been problems in earlier studies that used a method like this. The key to a randomization is that the patient be enrolled first, then randomized. If one knows ahead of time which rx will be assigned, can find reasons to exclude from your preferred r_x a patient that you don't think will do well.
With small n's as here, I would do more balancing i.e. constrain the randomization so as to give it a little help in balancing out the factors we know are important.

2 To demonstrate that there was compliance with the 'experimental' management. It is the numerical difference, not its "statistical" significance, that is the key here.

3 If any, a t-test for 2 independent samples. Could also use a rank sum test without much loss of power. Again, this is more a question of checking if the "process" was different in the 2 groups. Could use % reduction.

4 A t-test for 2 independent samples. Could also use a rank sum test .

5 If dealing with one group, finding 0 complications out of 21 doesn't mean that it would be 0/100 or 0/1000 (see JAMA article about zero numerators). Likewise, a comparison of complication rates in two small samples doesn't have much power (we don't have any events in either group, and statistical power, when dealing with proportions, is based on numbers of events. Think about calculating a confidence interval for the difference between, or ratio of, complication rates. Also, need longer follow-up.