The sign test

[first 2 paragraphs from A&B, §13.2 ; parts in sans serif font from JH. See also M&M Ch 5/7]

Suppose the observations in a sample of size n are x_1, x_2, . . ., x_n, and that of these r are positive and s negative. Some values of x may be exactly zero, and these would not be counted with either the positives or the negatives. The sum r + s may therefore be less than n, and will be denoted by n'. On the null hypothesis positive and negative values of x are equally likely.

Both r and s therefore follow a binomial distribution with parameters n' and π=1/2. Excessively high or low values of r (or equivalently, of s) can be tested exactly from tables of the binomial distribution....

We can use the π**=0.5** column in Table C of M&M and by ourselves add up the tail area from r onwards (and multiply by 2 for a 2-sided test). Or we can use the **Table entitled "SIGN TEST"** which I have prepared on a spreadsheet and included overleaf. The table shows the sum of the probabilities in one tail, so all one needs to do is multiply the entry by 2 for a two tailed p-value.

For large enough samples, the Gaussian approximation to the binomial can be used... with π=1/2, the distribution of r is symmetric and has $\mu = n' \bullet \pi$ and SD = $\sqrt{n' \pi [1-\pi]}$

Example(jh): In our example in §7.1, we worked out the case of r=8 "positives" among n'=10 non-zero differences.

By Table C, the probabilities of 8, 9 or 10 positive out of 10, when π=0.5, are

P-1tail = 0.0439 + 0.0098 + 0.0010 = 0.0547, so
P-2tail = 2 x 0.0547 = 0.1094.

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[Excel function: BINOMDIST(number_s, trials,probability_s, cumulative)
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Using the homegrown SIGN TEST Table, we locate the row marked n=10. The table is set up to handle the number of responses in the less frequent class, here the 2 'negatives' rather than the 8 positives. Since the Sign test is symmetric in r and s, we are therefore **interested in the tail consisting of the cumulation of 0, 1 and 2.** The entry in the column marked '2' gives us this cumulation... 0.055 (note everything in table is per 1000), the same as when the P-1tail = 0.0439 + 0.0098 + 0.0010 = 0.0547 above is rounded to 3 dp.

If we were to use the Gaussian distribution as an approximation, we would first calculate

$\mu[r] = n' \bullet \pi = 10(0.5) = 5,$ and $SD[r] = \sqrt{n' \times \pi \times \{1-\pi\}} = \sqrt{10 \times 0.5} \times 0.5 = 1.58.$

Thus, r=8 relative to this μ and SD is

$z = (8 - 5) / 1.58 = 1.90$ without the continuity correction

and

$z = \{|8 - 5| - 0.5\} / 1.58 = 1.58$ with the continuity correction

So

P-1tail = P(Z > 1.58) = 0.057,

remarkably close to the 'exact' calculation above.

Note that if one takes the formula

$$z = (r - r/2) / \sqrt{n \times 0.5} \times 0.5$$

and squares both sides, and uses the fact that r+s=n, one gets

$$z^2 = \frac{\{r - s\}^2}{r + s} \quad \text{or} \quad z^2 = \frac{\{|r - s| - 1\}^2}{r + s},$$

which 'saves on square roots' but must be referred to the chi-squared distribution with 1 degree of freedom.

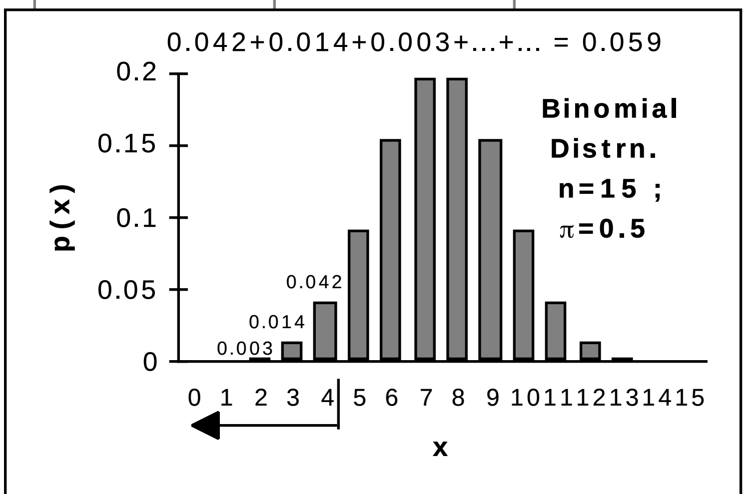
See link to test of proportions for paired data (McNemar Test) in Chapter 9.

SIGN TEST

Cumulative Binomial for $\pi = 0.5$; each entry to be read with a decimal point preceding it.

The 1-tail P value for a sign test is the cumulative probability corresponding to the number in the less frequent class
Double the table entry to get the 2-tailed P-value.

Example: A sign test with sample size n = 15 gives 11 positive observations and 4 negative observations. Thus, the number in the less frequent class is 4. Thus P = 0.059 (1-tailed) or 0.118 (2-tailed).



number in less frequent class

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	500																									
2	250																									
3	125	500																								
4	63	313																								
5	31	188	500																							
6	16	109	344																							
7	8	63	227	500																						
8	4	35	145	363																						
9	2	20	90	254	500																					
10	1	11	55	172	377																					
11		6	33	113	274	500																				
12		3	19	73	194	387																				
13		2	11	46	133	291	500																			
14		1	6	29	90	212	395																			
15			4	18	59	151	304	500																		
16			2	11	38	105	227	402																		
17			1	6	25	72	166	315	500																	
18			1	4	15	48	119	240	407																	
19				2	10	32	84	180	324	500																
20				1	6	21	58	132	252	412																
21				1	4	13	39	95	192	332	500															
22					2	8	26	67	143	262	416															
23					1	5	17	47	105	202	339	500														
24					1	3	11	32	76	154	271	419														
25						2	7	22	54	115	212	345	500													
26						1	5	14	38	84	163	279	423													
27						1	3	10	26	61	124	221	351	500												
28							2	6	18	44	92	172	286	425												
29							1	4	12	31	68	132	229	356	500											
30							1	3	8	21	49	100	181	292	428											
31								2	5	15	35	75	141	237	360	500										
32								1	4	10	25	55	108	189	298	430										
33								1	2	7	18	40	81	148	243	364	500									
34									1	5	12	29	61	115	196	304	432									
35									1	3	8	20	45	88	155	250	368	500								
36									1	2	6	14	33	66	121	203	309	434								
37										1	4	10	24	49	94	162	256	371	500							
38										1	3	7	17	36	72	128	209	314	436							
39										1	2	5	12	27	54	100	168	261	375	500						
40											1	3	8	19	40	77	134	215	318	437						
41											1	2	6	14	30	59	106	174	266	378	500					
42												1	4	10	22	44	82	140	220	322	439					
43												1	3	7	16	33	63	111	180	271	380	500				
44												1	2	5	11	24	48	87	146	226	326	440				
45													1	3	8	18	36	68	116	186	276	383	500			
46													1	2	6	13	27	52	92	151	231	329	441			
47													1	2	4	9	20	39	72	121	191	280	385	500		
48														1	3	7	15	30	56	97	156	235	333	443		
49														1	2	5	11	22	43	76	126	196	284	388	500	
50															1	3	8	16	32	59	101	161	240	336	444	556

Created in Excel™ using the BINOMDIST(successes,trials,0.5,cumulative) function

The textbook by Inglefinger, Mosteller, Thibodeau and Ware (we used their wording) has a similar table (Table III-9).