

## DISTRIBUTION OF WILCOXON RANK SUM STATISTIC: BY ENUMERATION

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[JH] In this table, purely for didactic purposes, I show all the equally likely configurations of the sum of the ranks in the smaller of 2 samples. Because of space, I could only fit in samples up to size  $n_1=3$  and  $n_2=5$ .

Take the example of  $n_1=3$  and  $n_2=5$  (bottom-most): there are 56 equally likely configurations of 3 ranks from the ranks 1-8, starting on the left with ranks 1-3 (if all 3 observations in sample 1 are smaller than all 5 observations in sample 2) to the opposite (6,7,8) on the extreme right. Thus the sum of the 3 ranks ranges from  $1+2+3=6$  to  $6+7+8=21$  [scale for the sum of the ranks is across the bottom and the top].

Thus, under  $H_0$ , there is a  $1/56$  chance of a sum of  $1+2+3=6$ , a  $1/56$  chance of  $1+2+4=7$ , a  $2/56$  chance of a sum of 8, etc., ..., a  $1/56$  chance of

$6+7+8=21$ . The 2-sided p-value corresponding to a sum of ranks of 6 is thus  $p(6) + p(21) = 1/56 + 1/56 = 2/56 = 0.038$ . Thus -- if my calculations are correct -- the sum of 6 should appear as significant at 0.05 (2-sided) at the beginning of Table A8 of Armitage and Berry.

Notice that no matter what the values of  $n_1$  and  $n_2$ , the distribution of the sum of ranks in the smaller sample is symmetric, and that it is already pretty close to Gaussian with very small  $n_1$  and  $n_2$ .

Notice also that if there are fewer than 40 equally likely configurations (e.g. with  $n_1=3$  and  $n_2=3$ , there are only 20), it is not even possible to have  $p(2\text{sided}) < 1/20 (= 0.05)$ .

**EXCERPTS FROM DISTRIBUTION OF WILCOXON RANK SUM STATISTIC**

n1	n2	Sum of Ranks in smaller sample																													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21									
1	1	1																													
1	2	1	2	3																											
1	3	1	2	3	4																										
2	2			1,2	1,3	2,3 1,4	2,4	3,4																							
2	3			1,2	1,3	2,3 1,4	2,4	3,4	2,5	3,5	4,5																				
2	4			1,2	1,3	2,3 1,4	2,4	3,4	2,5	3,5	4,5	3,4	2,6	3,6	4,6	5,6															
2	5			1,2	1,3	2,3 1,4	2,4	3,4	2,5	3,5	4,5	3,4	2,6	3,6	4,6	5,6	2,7	3,7	4,7	5,7	6,7										
2	6			1,2	1,3	2,3 1,4	2,4	3,4	2,5	3,5	4,5	3,4	2,6	3,6	4,6	5,6	2,7	3,7	4,7	5,7	6,7	7,8									
3	3					1,2,3	1,2,4	1,2,5	1,2,6	2,3,4	2,3,5	2,4,5	3,4,5	1,3,4	1,3,5	1,4,5	2,3,6	2,4,6	3,4,6	1,5,6	2,5,6	3,5,6	4,5,6								
3	4					1,2,3	1,2,4	1,2,5	1,2,6	2,3,4	2,3,5	2,4,5	3,4,5	1,3,4	1,3,5	1,3,6	1,4,6	1,5,6	2,4,7	2,5,7	3,5,7	4,5,7	1,6,7	2,6,7	3,6,7	4,6,7	5,6,7				
3	5					1,2,3	1,2,4	1,2,5	1,2,6	2,3,4	2,3,5	2,4,5	3,4,5	1,3,4	1,3,5	1,3,6	1,3,7	1,4,7	1,5,7	1,6,7	2,5,8	2,6,7	3,5,8	4,5,8	5,6,7	1,7,8	2,7,8	3,7,8	4,7,8	5,7,8	6,7,8

*JH 11/89H 11/89*