1. Let T be a positive random variable denoting the longevity of a randomly selected product, item, or person (such as an ink cartridge, battery, computer, iPod, or human). Denote the associated cumulative distribution function by $F_T(t)$, the survival function by $S_T(t) = 1 - F_T(t)$, the probability density function by $f_T(t)$, and the expectation $\int_0^\infty f_T(t)dt$ by μ_T. Show that

$$\mu_T = \int_0^\infty S_T(t)dt.$$

Heuristically: the mean longevity of 82.21 years in Fig. 1 is the total number of person-years (8221 P-Y) $\div$ the number of persons (100). The 8221 P-Y can be seen as the sum of the lengths of the horizontal lines (i.e., first sum the years for the same person, and then sum over persons) or the sum of the lengths of the vertical lines (i.e., first sum the persons for the same year of age, and then sum over years).

2. Consider the 'potential years of life lost' (PYLL) by a woman who dies of cervical cancer at age 45, i.e., the additional years she could have expected to live had she been protected against this cancer. Because the equation in (ii) may not be valid beyond 85, use the life-span 45-85 for both (i) and (ii).

Calculate (a) the (conditional) probability that a woman who reaches her 45th birthday will be alive on her 85th birthday, and (b) the expected (mean) number of additional years that women who reach 45 will live over the next 40 years. Determine (a) and (b) in 2 ways, based on the...

  i. 2000-2002 (Current) Complete Life-table, Canadian Females (next pages)

 ii. hazard rate / mortality rate / incidence density function[1], $h(age)$, fitted to the observed age-specific all-cause mortality rates for Québec women aged 45-85 in 2002:

$$\log h(age) = -6.7 + 0.10 \times (age - 45). \text{ see footnote}[2]$$

3. Calculate the not-for-profit[3] 1-year life-insurance premium for

  i. a Canadian woman aged 50, in "average" health, based on the 2000-2002 (Current) Complete Life-table for Canadian females;

 ii. a Québec woman aged 50, in "average" health, based on the fitted hazard function given above.

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[1]Gompertz (1779-1865) observed in 1825 that the force (intensity, I) of mortality at age a had the form $I_0\beta^a$ over a wide age-span i.e., age-specific death rates were log-linear-in-age (Gompertz 'Law of Mortality'). Random variables whose hazard functions follow this form are said to follow the Gompertz Distribution.

[2]Integral of this has closed form; or, could use num'l integration, e.g. `integrate` in R

[3]Such that in a large number of such insured persons, the premiums collected would just balance the total amount of the death benefits (each one valued at $10,000) paid out.
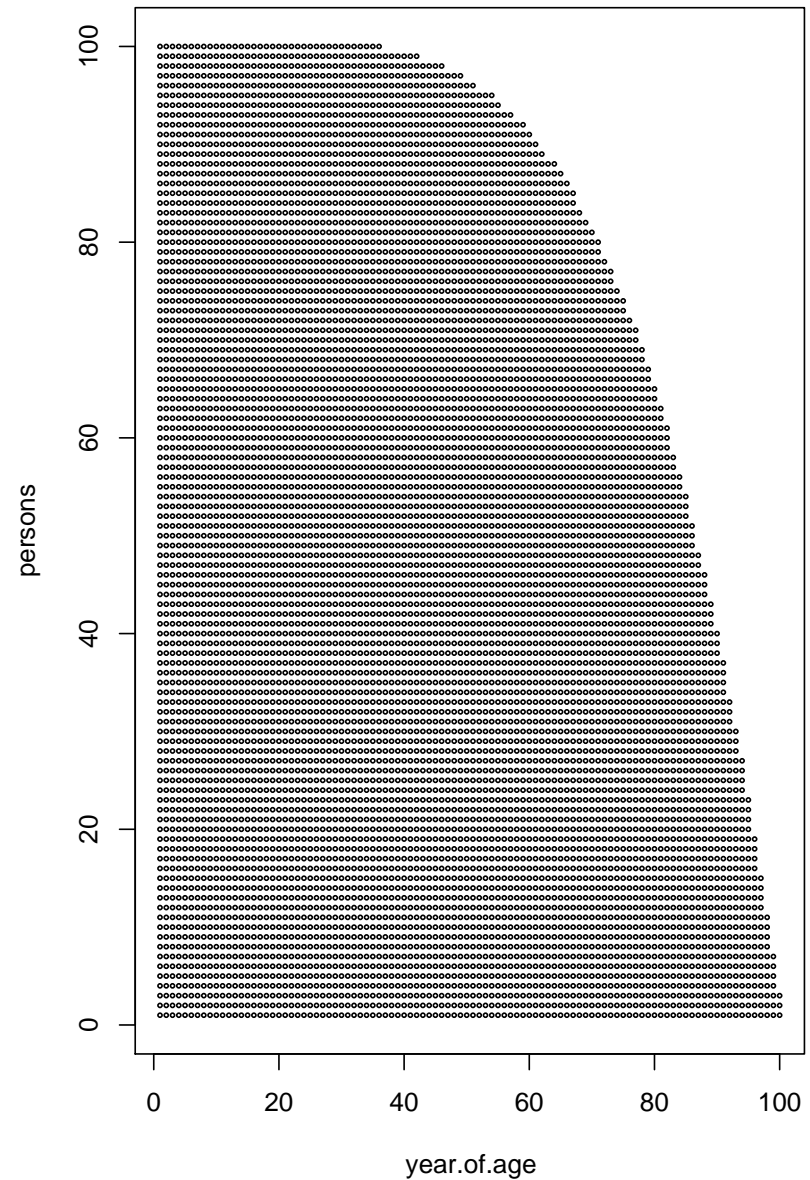

Figure 1: 8221 years lived by 100 persons

Table 2b Complete life table, Canada, 2000 to 2002: females

Age x	l_x	d_x	p_x	q_x	$cv(q_x)$	L_x	T_x	e_x	$cv(e_x)$
0 years	100000	467	0.99533	0.00467	2.1	99589	8203072	82.03	0.04
1 year	99533	35	0.99965	0.00035	7.5	99514	8103483	81.41	0.04
2 years	99498	20	0.99980	0.00020	9.8	99486	8003969	80.44	0.04
3 years	99478	15	0.99985	0.00015	11.2	99471	7904483	79.46	0.04
4 years	99463	12	0.99988	0.00012	12.1	99456	7805012	78.47	0.04
5 years	99451	10	0.99990	0.00010	18.7	99445	7705556	77.48	0.04
6 years	99441	9	0.99992	0.00008	25.9	99437	7606111	76.49	0.04
7 years	99432	7	0.99993	0.00007	26.6	99429	7506674	75.50	0.04
8 years	99425	7	0.99993	0.00007	25.1	99421	7407245	74.50	0.04
9 years	99418	7	0.99993	0.00007	23.0	99415	7307824	73.51	0.04
10 years	99411	9	0.99991	0.00009	21.2	99407	7208409	72.51	0.04
11 years	99402	8	0.99991	0.00009	20.2	99398	7109002	71.52	0.04
12 years	99394	13	0.99987	0.00013	19.8	99387	7009604	70.52	0.04
13 years	99381	15	0.99984	0.00016	15.1	99374	6910217	69.53	0.04
14 years	99366	20	0.99980	0.00020	12.1	99356	6810843	68.54	0.04
15 years	99346	24	0.99976	0.00024	12.0	99333	6711487	67.56	0.04
16 years	99322	28	0.99972	0.00028	12.7	99308	6612154	66.57	0.05
17 years	99294	31	0.99969	0.00031	12.5	99278	6512846	65.59	0.05
18 years	99263	33	0.99967	0.00033	11.0	99246	6413568	64.61	0.05
19 years	99230	33	0.99966	0.00034	9.6	99214	6314322	63.63	0.05
20 years	99197	34	0.99966	0.00034	9.7	99180	6215108	62.65	0.05
21 years	99163	33	0.99966	0.00034	11.2	99146	6115928	61.68	0.05
22 years	99130	33	0.99966	0.00034	12.0	99114	6016782	60.70	0.05
23 years	99097	33	0.99967	0.00033	11.2	99080	5917668	59.72	0.05
24 years	99064	33	0.99967	0.00033	10.0	99047	5818588	58.74	0.05
25 years	99031	32	0.99967	0.00033	10.2	99015	5719541	57.76	0.05
26 years	98999	33	0.99967	0.00033	11.5	98982	5620526	56.77	0.05
27 years	98966	33	0.99967	0.00033	12.1	98950	5521544	55.79	0.05
28 years	98933	34	0.99965	0.00035	10.9	98916	5422594	54.81	0.05
29 years	98899	36	0.99963	0.00037	9.4	98881	5323678	53.83	0.05
30 years	98863	39	0.99961	0.00039	9.3	98843	5224797	52.85	0.06
31 years	98824	42	0.99958	0.00042	10.0	98803	5125954	51.87	0.06
32 years	98782	45	0.99954	0.00046	9.9	98760	5027151	50.89	0.06
33 years	98737	50	0.99950	0.00050	8.5	98711	4928391	49.91	0.06
34 years	98687	54	0.99945	0.00055	7.1	98660	4829680	48.94	0.06
35 years	98633	60	0.99939	0.00061	6.9	98603	4731020	47.97	0.06
36 years	98573	66	0.99933	0.00067	7.3	98539	4632417	46.99	0.06
37 years	98507	72	0.99927	0.00073	7.2	98471	4533878	46.03	0.06
38 years	98435	78	0.99921	0.00079	6.2	98397	4435407	45.06	0.06
39 years	98357	83	0.99915	0.00085	5.4	98315	4337010	44.09	0.07

Table 2b Complete life table, Canada, 2000 to 2002: females

Age x	l_x	d_x	p_x	q_x	$cv(q_x)$	L_x	T_x	e_x	$cv(e_x)$
40 years	98274	91	0.99908	0.00092	5.5	98229	4238695	43.13	0.07
41 years	98183	97	0.99901	0.00099	5.9	98134	4140466	42.17	0.07
42 years	98086	107	0.99891	0.00109	5.9	98033	4042332	41.21	0.07
43 years	97979	117	0.99880	0.00120	5.0	97920	3944299	40.26	0.07
44 years	97862	129	0.99868	0.00132	4.4	97797	3846379	39.30	0.07
45 years	97733	142	0.99855	0.00145	4.5	97662	3748582	38.36	0.07
46 years	97591	156	0.99840	0.00160	5.0	97513	3650920	37.41	0.08
47 years	97435	171	0.99824	0.00176	4.9	97350	3553407	36.47	0.08
48 years	97264	187	0.99807	0.00193	4.2	97170	3456057	35.53	0.08
49 years	97077	204	0.99790	0.00210	3.7	96975	3358887	34.60	0.08
50 years	96873	222	0.99771	0.00229	3.8	96761	3261912	33.67	0.08
51 years	96651	243	0.99749	0.00251	4.2	96530	3165151	32.75	0.08
52 years	96408	266	0.99724	0.00276	4.1	96275	3068621	31.83	0.09
53 years	96142	293	0.99695	0.00305	3.5	95996	2972346	30.92	0.09
54 years	95849	323	0.99663	0.00337	3.2	95687	2876350	30.01	0.09
55 years	95526	355	0.99628	0.00372	3.4	95349	2780663	29.11	0.09
56 years	95171	390	0.99590	0.00410	3.7	94976	2685314	28.22	0.10
57 years	94781	427	0.99549	0.00451	3.7	94568	2590338	27.33	0.10
58 years	94354	466	0.99506	0.00494	3.2	94121	2495770	26.45	0.10
59 years	93888	505	0.99462	0.00538	2.8	93636	2401649	25.58	0.10
60 years	93383	548	0.99413	0.00587	3.0	93109	2308013	24.72	0.11
61 years	92835	595	0.99359	0.00641	3.3	92538	2214904	23.86	0.11
62 years	92240	649	0.99296	0.00704	3.3	91915	2122366	23.01	0.11
63 years	91591	709	0.99226	0.00774	2.8	91236	2030451	22.17	0.11
64 years	90882	772	0.99150	0.00850	2.4	90496	1939215	21.34	0.12
65 years	90110	841	0.99067	0.00933	2.5	89689	1848719	20.52	0.12
66 years	89269	915	0.98975	0.01026	2.8	88812	1759030	19.70	0.13
67 years	88354	999	0.98869	0.01131	2.7	87854	1670218	18.90	0.13
68 years	87355	1086	0.98757	0.01243	2.3	86812	1582364	18.11	0.13
69 years	86269	1175	0.98638	0.01362	2.0	85682	1495552	17.34	0.14
70 years	85094	1271	0.98507	0.01493	2.1	84458	1409870	16.57	0.14
71 years	83823	1378	0.98355	0.01645	2.3	83134	1325412	15.81	0.15
72 years	82445	1503	0.98177	0.01823	2.2	81694	1242278	15.07	0.15
73 years	80942	1635	0.97981	0.02019	1.9	80124	1160584	14.34	0.16
74 years	79307	1768	0.97770	0.02230	1.6	78423	1080460	13.62	0.16
75 years	77539	1913	0.97533	0.02467	1.7	76582	1002037	12.92	0.17
76 years	75626	2074	0.97258	0.02742	1.9	74589	925455	12.24	0.18
77 years	73552	2255	0.96934	0.03066	1.8	72425	850866	11.57	0.19
78 years	71297	2441	0.96576	0.03424	1.5	70076	778441	10.92	0.20
79 years	68856	2621	0.96193	0.03807	1.4	67546	708365	10.29	0.21

Table 2b Complete life table, Canada, 2000 to 2002: females

Age x	l_x	d_x	p_x	q_x	$cv(q_x)$	L_x	T_x	e_x	$cv(e_x)$
80 years	66235	2809	0.95760	0.04240	1.6	64830	640819	9.67	0.23
81 years	63426	3011	0.95252	0.04748	1.7	61920	575989	9.08	0.24
82 years	60415	3235	0.94646	0.05354	1.6	58798	514069	8.51	0.26
83 years	57180	3470	0.93932	0.06068	1.4	55445	455271	7.96	0.27
84 years	53710	3691	0.93128	0.06872	1.3	51865	399826	7.44	0.29
85 years	50019	3879	0.92245	0.07755	1.4	48080	347961	6.96	0.32
86 years	46140	4015	0.91297	0.08703	1.6	44132	299881	6.50	0.35
87 years	42125	4088	0.90296	0.09704	1.5	40081	255749	6.07	0.38
88 years	38037	4095	0.89233	0.10767	1.3	35990	215668	5.67	0.41
89 years	33942	4039	0.88101	0.11899	1.4	31922	179678	5.29	0.46
90 years	29903	3914	0.86912	0.13088	1.7	27946	147756	4.94	0.52
91 years	25989	3722	0.85678	0.14322	1.9	24128	119810	4.61	0.57
92 years	22267	3471	0.84412	0.15588	1.8	20532	95682	4.30	0.63
93 years	18796	3212	0.82913	0.17087	1.8	17190	75150	4.00	0.71
94 years	15584	2911	0.81320	0.18680	2.0	14129	57960	3.72	0.81
95 years	12673	2582	0.79624	0.20376	2.1	11382	43831	3.46	0.94
96 years	10091	2238	0.77823	0.22177	2.4	8972	32449	3.22	1.10
97 years	7853	1891	0.75917	0.24083	2.6	6908	23477	2.99	1.30
98 years	5962	1556	0.73906	0.26094	3.0	5184	16569	2.78	1.55
99 years	4406	1243	0.71791	0.28209	3.4	3784	11385	2.58	1.87
100 years	3163	962	0.69575	0.30425	3.9	2682	7601	2.40	2.30
101 years	2201	721	0.67260	0.32740	4.7	1841	4919	2.23	2.90
102 years	1480	520	0.64849	0.35151	5.5	1220	3078	2.08	3.66
103 years	960	361	0.62349	0.37651	6.9	779	1858	1.94	4.75
104 years	599	241	0.59763	0.40237	8.3	478	1079	1.80	6.19
105 years	358	154	0.57098	0.42902	11.1	281	601	1.68	8.40
106 years	204	93	0.54362	0.45638	14.4	158	320	1.57	11.31
107 years	111	54	0.51561	0.48439	17.6	84	162	1.46	15.35
108 years	57	29	0.48704	0.51296	26.4	43	78	1.36	22.55
109 years	28	15	0.45800	0.54200	37.1	20	35	1.27	31.18

Note: Estimates with a coefficient of variation (cv) greater than 33.3% are to be used with caution
F   too unreliable to be published (indicates a cv of at least 100.0%).