Link b/w Poisson (Counting) Distrⁿ & Exponentially Distributed Waiting Times

width of time (or Distance) window $= w$

Ave (wait) = ave (distance between events) $= \mu_T$ (T = "time")

Y: pdf $f(y) = \frac{1}{\mu_T} e^{-y/\mu_T}$

$Pr(0 \text{ 'events'}) = Prob(Y > w) = cdf(w) = e^{-\frac{1}{\mu_T} \cdot w} =$ PoissonProb$(0, \text{"}\mu_c\text{"} = w/\mu_T$

c = "count"

$Pr(1 \text{ 'event'}) = Pr[1^{st} \text{ event somewhere in } (0, w)] \cdot Pr[\text{no further event}]$

$$= \int_0^w pdf(s) \cdot e^{-[w-s]/\mu_T} \, ds$$

$$= \int \frac{1}{\mu_T} e^{-s/\mu} \cdot e^{-\frac{w-s}{\mu}} ds = \frac{1}{\mu} e^{-\frac{w}{\mu}} \int_0^w ds = \frac{w}{\mu} e^{-\frac{w}{\mu}}$$

$=$ PoissonProb$\left(c=1, \mu_c = \frac{w}{\mu_T}\right)$

etc etc etc