The equivalence of cost-effectiveness analysis (CEA) and cost-benefit analysis (CBA) has been vigorously debated in the health economic literature. In this paper we review and refine the conditions for the equivalence of CEA and CBA. The previously stated conditions require that 1) each individual’s willingness to pay (WTP) per quality-adjusted life year (QALY) is constant and does not vary with the magnitude of QALY gains, and 2) the WTP per QALY is identical for every individual in society. Based on mathematical programming formulations of CEA and CBA, we note that condition 2 can be replaced with two other conditions, which together are less restrictive than the requirement that every individual have the same WTP per QALY. Even with this less restrictive set of conditions, CEA and CBA are unlikely to be equivalent under real world conditions. When CEA and CBA do lead to different resource allocation decisions, the most appropriate framework for health economic analysis depends on the perspective regarding distribution issues. We also examine the equivalence of two different definitions of CEA provided in the literature and discuss the problems that could arise when there are multiple optima.

Keywords: cost-benefit analysis, cost-effectiveness analysis.

Introduction

The distinctions between cost-effectiveness analysis (CEA) and cost-benefit analysis (CBA) have been vigorously debated in the health economic literature [1,2]. The metric for decision making in CEA is the ratio of the incremental cost to incremental effectiveness, measured in quality-adjusted life years (QALYs). On the other hand, the metric in CBA is the net benefit, defined as the difference between the incremental benefits and incremental costs.

Phelps and Mushlin [1] argue that the two analyses are equivalent and that decisions made about medical resources using CE analysis are wholly analogous to those using CB analysis. This implies that analyses using CEA and CBA should result in identical allocation of health-care resources and that the only distinction between CEA and CBA is in the measurement of health benefits.

An alternate approach to examining the distinction between CEA and CBA was provided by Donaldson [2] and was based on the question that the analyst was attempting to answer, rather than what was being measured. According to his definition, the objective of CEA is to determine the least costly way to achieve a goal, while the objective of CBA is to determine whether the goal is worth achieving. Based on these definitions, he argues that the near equivalence between the two methodologies is a fallacy, since they do not address the same allocation question.

A third point of view on the distinction between CEA and CBA was provided by Johannesson [3], in which he argued that CEA could be viewed as a subset of CBA. Furthermore, CEA and CBA would be equivalent if: 1) the willingness to pay (WTP) per unit of effectiveness is constant; and 2) this WTP is the same for everyone.

In this paper, we attempt to shed some light on these seemingly contradictory assertions in the health economic literature. In reality, the alternative approaches described above overlap substantially and differ from one another mostly in the definitions of CEA and CBA. The objectives of the paper are:

1. to formulate CEA and CBA as optimization problems using a mathematical programming framework;
2. to derive conditions under which the different formulations are equivalent;
3. to examine whether these conditions of equivalence are likely to hold under real-world conditions;
4. to examine the choice of the most appropriate methodology for health economic analysis when CEA and CBA are not equivalent.

Definitions of CEA and CBA
In this section, we will focus on allocation of resources for health interventions. We derive formulations for CEA and CBA from the perspective of a societal decision maker whose task it is to choose the optimal set of health-care interventions that will maximize societal well being. Ideally, such allocation of health-care resources will occur in the context of allocating the entire societal budget, where decisions regarding not only health-care spending, but also regarding allocation of resources for education, the environment and public works, have to be made. We will address health economic analysis and its focus on allocation of resources for health interventions and will refer to the problem of societal resource allocation only when it has a bearing on the choice of methodology for health economic analysis.

We first define “I” potentially overlapping subgroups within a given society. Individual members of a subgroup are assumed to suffer from the same medical condition, to be in identical health states, with the same expected response to treatments for that condition and of the same socioeconomic category. This does not imply that all individuals in a subgroup have identical response to treatment, or identical preferences for health outcomes. It implies only that though individual responses to treatment may vary, characteristics such as age, gender, disease duration, and comorbidity are not used as distinguishing features between members of the subgroup in terms of their expected response to available treatments, or the willingness to pay for improvements in health outcomes. Note that one individual can belong to more than one subgroup if he/she suffers from more than one medical condition.

Let J(i) be the finite set of medical interventions available to treat individuals in subgroup i. If two drugs, say A and B, and their combination (A + B) are available to treat a given subgroup, these would be considered to be three different interventions. Similarly, different dosages of the same drug would be considered to be different interventions. Note that the absence of, or no treatment, is an option that should be included in J(i) for any subgroup i. The objective of the societal decision maker is to assign one and only one intervention from J(i) to each individual in a subgroup i. The framework is such that the decision maker can assign different interventions for different individuals within a given subgroup. It may not be logical for the societal decision maker to treat certain patients in a subgroup differently from others given that the individuals are indistinguishable in terms of their expected response to each intervention. However, this could indeed happen for certain formulations of the CEA problem, as will be demonstrated.

We further define the variables used to formulate CEA and CBA as optimization problems. Let N_i be the number of individuals in subgroup i. In view of the decision maker’s objective to assign each patient in a subgroup to one and only one intervention, the decision maker must determine the number of individuals N_i (0 ≤ N_i ≤ N_o) in subgroup i to be treated with each intervention j (∈ J(i)), such that Σ j∈J o N_j = N_i. This constraint ensures that each patient is treated with one and only one intervention. The allocation of individuals to different interventions in CEA depends on the QALY gain (Q_ij) and cost (C_ij) for a random individual in subgroup i treated with intervention j. In the CBA framework, the benefit provided by the intervention is measured not only in terms of QALYs, but also in terms of the WTP (W_ij) for intervention j, of a random individual in subgroup i. One can use the monetary amount that an individual is willing to pay per QALY to examine the equivalence of CEA and CBA. Let v be the WTP per QALY of a randomly chosen individual in the society and v_i the WTP per QALY of a randomly chosen individual in subgroup i. The distribution of v_i could differ from that of v if the subgroup i is different from the overall population in terms of WTP per QALY. Note that the difference between W_o and v is that W_o is the WTP for the QALY gain from intervention j, while v_i is the WTP for one QALY.

Of the two methodologies under discussion, CEA is more widely used in health economic analysis. The CEA decision rule as described by Johannesson and Weinstein [4] is to treat every individual in each subgroup with the intervention that has the highest incremental cost-effectiveness ratio below a threshold value V, after eliminating interventions ruled out because of simple or extended dominance. The threshold value V corresponds to the monetary value that society places on a unit of effectiveness or one QALY, for instance. In this framework the objective of the decision maker is to allocate...
resources in a manner that will maximize QALYs with the condition that each additional QALY cannot cost more than V dollars. To estimate the value of V, the societal decision maker needs to determine what society is willing to pay for each additional QALY. Even though there are several potential sources of V [5], welfare economic theory suggests that V should be based on WTP per QALY of the individuals in that society.

At this point, it is important to reiterate the difference between v, v_i, and V for the sake of clarity. All three measure the WTP for one QALY. As stated, v is the WTP per QALY of a randomly chosen individual in the society, and v_i the WTP per QALY of a randomly chosen individual in subgroup i. Thus, E[v] = E[v_i] for all i, if each subgroup is identical in terms of WTP per QALY. V is the monetary value that a societal decision maker places on one QALY, and is commonly set equal to the expected value of WTP per QALY of the individuals in the society ⟨E[v]⟩ [3].

CEA as defined by Johannesson and Weinstein [4] is now presented as an optimization problem. Laska et al. [6], proved mathematically that Johannesson and Weinstein’s [4] CEA decision rule is identical to a decision rule for choosing the intervention that maximizes net benefit for each disease, where the value V is used to convert expected QALY gains into monetary units. As per Laska et al. [6], the decision maker can solve the following linear program to determine the number of patients in each subgroup that should be treated with the different interventions such that QALYs are maximized, while paying no more than V dollars for each additional QALY.

CEA-V:Max ∑_{j∈J(i)} (V × E[Q_{ij}] - E[C_{ij}]) × N_{ij} (1)

ST: ∑_{j∈J(i)} N_{ij} = N_i, ∀i (2)

where E[Q_{ij}] and E[C_{ij}] are the expected QALY gain and expected cost per person, respectively, for individuals in subgroup i treated with intervention j.

As noted earlier, it is interesting to consider at this point whether the societal decision maker would treat some patients in a subgroup differently than others. In fact, it is easy to show that there exists an optimal solution to CEA-V such that N_{ij} = N_i for one j* ∈ J(i) and N_{ij} = 0 for all j' ≠ j*, for every subgroup i (see Appendix A for the proof). In other words, there exists at least one optimal solution to CEA-V where every individual in each subgroup is treated with one and only one intervention from the available options. This intervention j* is the one that maximizes (V × E[Q_{ij}] - E[C_{ij}]) for the given i. Note that alternate optimal solutions may exist in which some proportion of individuals in a subgroup may be treated with one intervention, and the rest with another. This happens when more than one intervention has the same maximum value for (V × E[Q_{ij}] - E[C_{ij}]). This will be examined more closely in the next section.

The CEA-V formulation does not have an explicit budget constraint and hence, does not place an upper bound on the amount of money that can be spent on health care. The budget constraint is implicitly captured through the value V, which places an upper bound on the amount of money that society is willing to spend to increase health by one QALY.

An alternate definition of CEA was provided by Donaldson [2]. In his definition of CEA, the objective of the decision maker is to maximize effectiveness while holding costs within a given health-care budget. Let us assume that an amount of B dollars is available in the health-care budget. It may seem odd to prespecify a budget independent of the cost of available treatments, although decision makers may face such budget constraints, at least in the short term. The decision maker can solve the following linear program to determine the number of patients in each subgroup that should be treated with the different interventions to maximize QALYs while holding costs within the given health-care budget:

CEA-B:Max ∑_{j∈J(i)} (E[Q_{ij}] × N_{ij}) (3)

ST: ∑_{j∈J(i)} (E[C_{ij}] × N_{ij}) ≤ B (4)

∑_{j∈J(i)} N_{ij} = N_i, ∀i (5)

In contrast to CEA-V, which does not place an explicit limit on health-care expenditures, in CEA-B the health-care expenditures are limited to be less than B dollars. In contrast to CEA-V, it is possible that in the CEA-B formulation the only optimal solution may involve treating different individuals in a given subgroup with different interventions (Appendix 1 # 2). The equity issues raised by treating different individuals in a subgroup with different interventions has been addressed elsewhere [7]. Distinctions between CEA-V and CEA-B are discussed in more detail below.

CBA is a commonly used analytical framework for economic evaluations. In the CBA framework, the objective of the decision maker is to allocate resources to maximize the expected societal net benefit, which is the difference between the
Equivalence of CEA and CBA

expected WTP for the interventions and the expected cost of providing the interventions. Based on the principles of welfare economics, the decision maker estimates the societal net benefit as an aggregation of individual net benefits. The decision maker can solve the following linear program to determine the number of patients in each subgroup that should be treated with the different interventions to maximize expected societal net benefit:

\[
\text{CBA: Max } \sum_i \sum_{j|i} (E[W_{ij}] - E[C_{ij}]) \times N_{ij}
\]  

(6)

\[
\sum_{i} N_{ij} = N_i, \quad \forall i
\]  

(7)

where \(E[W_{ij}]\) is the expected willingness to pay for intervention \(j\) for individuals in subgroup \(i\).

In the following section, we compare CBA to CEA-V, which is the commonly used formulation for CEA. Thereafter, we compare the two different formulations of CEA: CEA-V and CEA-B.

CEA-V versus CBA

The conditions for equivalence of CEA-V and CBA based on objective functions (1) and (6) above are derived as follows: the two formulations would be equivalent if

\[
V \times E[Q_{ij}] = E[W_{ij}]
\]  

(8)

Johannesson [3] states the two conditions that are required for the equivalence of CEA and CBA as follows:

1. the WTP per QALY is constant for an individual, i.e., does not vary with the magnitude of QALY gain or loss;
2. the WTP per QALY is the same for all individuals.

If condition (1) holds, then WTP for intervention \(j\) for an individual in subgroup \(i\) is given by:

\[
W_{ij} = v_i \times Q_{ij}
\]  

(9)

Since the individual is willing to pay \(v_i\) for every QALY gained, regardless of the magnitude of QALY gain. If condition (2) holds, then \(v_i\) and \(V\) are both equal to the same constant. If both conditions (1) and (2) above hold, then:

\[
E[W_{ij}] = E[v_i \times Q_{ij}] \quad (\text{based on condition 1})
\]  

(10)

\[
= V \times E[Q_{ij}] \quad (v_i = V \text{ based on condition 2})
\]  

(11)

Thus, it is evident that the conditions stated by Johannesson [3] are sufficient for the equivalence of CEA and CBA.

However, Johannesson’s condition (2) is not necessary and can be replaced by the following two conditions, which together are less restrictive than condition (2):

2a) the expected WTP per QALY is the same in every subgroup, and

2b) the WTP per QALY (\(v_i\)) and the QALY gain from an intervention \(j\) (\(Q_{ij}\)) for a randomly chosen individual in subgroup \(i\) are independent random variables.

Condition 2a implies that:

\[
V = E[v_i], \quad \forall i.
\]  

(13)

Johannesson [3] illustrated using an example of how the equivalence of CEA and CBA may be violated when the WTP per QALY varies across subgroups. Condition 2b implies that:

\[
E[v_i \times Q_{ij}] = E[v_i] \times E[Q_{ij}].
\]  

(13)

If conditions (1) (2a), and (2b) hold then:

\[
E[W_{ij}] = E[v_i \times Q_{ij}] \quad (\text{based on condition 1})
\]  

(14)

\[
= E[v_i] \times E[Q_{ij}] \quad (\text{based on condition 2b})
\]  

(15)

\[
= V \times E[Q_{ij}] \quad (\text{based on condition 2a})
\]  

(16)

Thus, if conditions (1) (2a), and (2b) hold, CEA and CBA are equivalent. Conditions 2a and 2b together are less restrictive than condition (2), which requires that the WTP per QALY be identical for every individual. Conditions (2a) and (2b) taken together do not require this. Instead they require only that the expected WTP per QALY be the same in every subgroup, and that \(v_i\) and \(Q_{ij}\) be independent within a given subgroup \(i\). The independence implies that the value of one of the variables does not influence the value of the other. Thus, the WTP per QALY of an individual has no influence on the QALY gain derived by that individual from a given intervention. Empirical data is not currently available to determine whether this is a reasonable assumption.

Reasons for Divergence

The assumption that WTP per QALY is constant within an individual (condition 1) is not realistic; an individual’s WTP per QALY will generally decrease with increases in QALY’s. Such a relationship between WTP per QALY and magnitude of QALY gain for an individual would violate condition (1) required for the equivalence of CEA-V and CBA. Hence, the resource allocations derived
from CEA-V and CBA will not be identical in practice.

Even if the WTP per QALY was constant at the individual level, we have shown that two more conditions need to be satisfied for CEA-V and CBA to be equivalent. First, the expected WTP per QALY must be the same for all subgroups. If some subgroups, such as those with diseases resulting from malnutrition or lead poisoning are composed of primarily lower-income individuals, the equality of WTP per QALY across subgroups may not hold true. Second, \( v_i \) and \( Q_{ij} \) need to be independent within a given subgroup. An association between an individual’s WTP per QALY and their QALY-gain may exist. Such an association would violate the assumption of independence of \( v_i \) and \( Q_{ij} \). Further empirical studies are needed to test whether condition 2b holds true in practice.

**CEA or CBA**

The conditions for equivalence of CEA-V and CBA are unlikely to hold in practice. The logical next question is which of these methodologies is preferable if recommendations for resource allocation afforded by CEA-V and CBA diverge?

Some contend that CBA has a closer and more established connection to welfare economics than CEA, and has an explicit grounding in welfare economic principle [8]. If one accepts welfare economics as the foundation for the economic analysis, CBA would be the methodology of choice.

The welfare economic foundation of health-care decision making has been criticized by economists for ignoring distributional issues [9]. The main criticism that has been raised regarding this approach is that it could lead to health-care resources being distributed away from individuals who place a relatively low value on health, potentially due to their inability to pay for it [10]. An alternative to the welfarist approach is the extra-welfarist model in which the value of a unit of health such as a QALY is identical for every individual in society, and one QALY is assumed to provide the same benefit or value to each individual. Valuation of health benefits in the extra-welfarist approach is necessarily based on individual valuations of these benefits. Since CBA is firmly rooted in welfare economics, acceptance of the extra-welfarist principles would favor the use of CEA-V, as it allows decision makers to value health gains independently of the current distribution of wealth, which is at the root of the distinction between the welfarist and extra-welfarist approaches.

**CEA-V versus CEA-B**

The contrast between the two CEA formulations: CEA-V and CEA-B was examined by Donaldson [2]. CEA-V examines how to allocate health-care resources to maximize net societal benefit based on the value \( V \) that society is willing to pay for a unit of effectiveness; CEA-B attempts to maximize the QALY gain that can be achieved within a given health-care budget. Thus, CEA-B does not ask whether the specified health-care budget is sufficient or excessive; instead, it considers the budget to be exogenous to the decision problem. In CEA-V, the health-care budget is not prespecified but is derived as part of the solution. The fact that resources are limited is implicit in the monetary value (\( V \)) placed on the unit of effectiveness.

There are also conditions under which the two formulations, CEA-V and CEA-B, yield identical solutions (\( N_{ij}^* \)). Using the Lagrangian multiplier \( \lambda \) to relax the cost constraint (4) in formulation CEA-B leads to the following formulation:

\[
\text{CEA-B'}: \max \{ \Sigma \Sigma_{i,j,i,j} (E[Q_{ij}] \times N_{ij}) \} + \lambda \left[ B - \Sigma \Sigma_{i,j,i,j} (E[C_{ij}] \times N_{ij}) \right]
\]

\[
\text{ST: } N_{ij} = N_{i}, \text{ } \forall \text{ I}
\]

If \( (\lambda^*,N_{ij}^*) \) is an optimal solution to CEA-B', then \( N_{ij}^* \) is an optimal solution to CEA-B. The optimal \( \lambda^* \) in CEA-B' corresponds to the shadow price of the budget constraint. This shadow price represents change in the maximized total effectiveness for a unit change in the budget. Thus, if the budget \( B \) is increased by a dollar, maximized total effectiveness increases by \( \lambda^* \).

If \( \lambda \) is set equal to its optimum value \( \lambda^* \) in CEA-B', \( N_{ij}^* \) is an optimal solution to the resulting formulation. If we substitute \( \lambda^* \) for \( \lambda \) and rearrange terms, the objective function (17) can be written as follows:

\[
\{ \Sigma \Sigma_{i,j,i,j} (E[Q_{ij}] - \lambda^* \times E[C_{ij}]) \times N_{ij} \} + \lambda^* \times B
\]

Multiplying the objective function by a constant, and eliminating terms that are constants does not change the optimal solution. Multiplying (19) by \( 1/\lambda^* \) and eliminating the constant \( B \) gives the following objective function:

\[
\Sigma \Sigma_{i,j,i,j} \left( \frac{E[Q_{ij}]}{\lambda^*} - E[C_{ij}] \right) \times N_{ij}
\]

If \( V = 1/\lambda^* \), the formulations for CEA-B' and CEA-V are identical. Thus, CEA-V and CEA-B have at least one optimal solution in common, assuming that the monetary value of a unit of effectiveness in the CEA-V formulation corresponds to the recipro-
Equivalent of CEA and CBA

Although this result was also been noted by others [5], the issue of multiple optima has not been discussed in detail in the health economic literature. As demonstrated, even when \( V = 1/\lambda^* \), when multiple optima exist for CEA-V, some of these optimal solutions may violate the budget constraint in CEA-B. As noted, CEA-V always has an optimal solution, which involves treating everyone in a subgroup with the same intervention, while this is not necessarily true for CEA-B. Let us consider the case where the optimal solution for CEA-B involves treating a proportion \( \beta^* \) of individuals in subgroup \( i_1 \) with intervention \( j_1 \) and the rest of the individuals \((1 - \beta^*)\) with intervention \( j_2 \). When \( V = 1/\lambda^* \) is used in CEA-V there are as many optima as there are patients in subgroup \( i_1 \), with a proportion \( \beta \) \((0 \leq \beta \leq 1)\) of individuals in subgroup \( i_1 \) receiving intervention \( j_1 \) and proportion \( 1-\beta \) receiving intervention \( j_2 \). This happens because

\[
V \times E[Q_{i1}] - E[C_{i1}] = V \times E[Q_{i2}] - E[C_{i2}]
\]

and the line segment joining \( j_1 \) and \( j_2 \) has a slope of \( V \) as demonstrated in Appendix A. If \( E[C_{i1j1}] > E(C_{i1j2}) \), then solutions where \( \beta > \beta^* \) will violate the budget constraint.

The issue of multiple optima can be clearly illustrated using the graphical approach for optimal resource allocation with CEA-V and CEA-B. The graphical method makes use of the cost-effectiveness frontier (CEF) to determine the optimal solution. The CEF connects nondominated interventions by line segments in order of increasing effectiveness.

Figure 1 is an example of a smooth convex CEF curve. The slope of the tangent to the CEF at any point is the incremental cost of achieving an increase in effectiveness of one unit. As one moves away from the origin, the total societal health benefits and the health-care expenditure increase. However the marginal cost of achieving a unit gain in effectiveness also increases with the slope of the tangent. The optimal solution to the CEA-B corresponds to the point (Y) on the CEF where the per-person cost is \( B_i/N_i \), where \( B_i \) is the hypothetical budget allocated for treating subgroup \( i \). This methodology is easily extended to allocate resources for all subgroups based on an overall budget \( B \). The shadow price \( (\lambda^*) \) is equal to the reciprocal of the slope of the tangent to the CEF at Y, as the reciprocal of the slope gives the marginal change in effectiveness for a unit change in the expenditure. The optimal solution to the CEA-V formulation corresponds to the point of tangency (Z) between a line of slope \( V \) and the CEF and must equal \( 1/\lambda^* \) for resource allocation to be identical for CEA-V and CEA-B.

In practice, the CEF may not be a smooth curve because a limited number of health interventions are considered. In this case, the CEF is piecewise linear as in Figure 2. The line of slope \( V \) tangential to the CEF could pass through one of the interventions or overlap with one of the line segments of the CEF, as in the figure. In either case, the objective function can be maximized by treating all patients with an intervention that lies on the tangent.

In the case for CEA-B where the horizontal line through \( B_i/N_i \) intersects the CEF between interventions L and M, the optimal solution corresponds to treating some patients with L and others with M. If \( V \) is set equal to \( 1/\lambda^* \), all points on the line segment between interventions L and M are optimal solutions for CEA-V. These points correspond to providing interventions L and M to different proportion of patients in the subgroup such that the proportions sum to 1. However, the solutions to CEA-V that lie above Y on line segment LM violate the budget constraint, and are infeasible under CEA-B.

**Reasons for Divergence**

If policymakers were to equate \( V \) and \( 1/\lambda^* \), CEA-V and CEA-B could lead to identical allocation decisions, although when CEA-V has alternate optima, only one of these solutions corresponds to the optimal solution for CEA-B. In theory, there are two means of achieving equality of \( V \) and \( 1/\lambda^* \). The decision maker could either set the value \( V \) in the CEA-V equal to \( 1/\lambda^* \) or choose \( B \) so that \( 1/\lambda^* \) is equal to a predefined societal value of \( V \). Under either alternative, the decision maker needs to solve.

![Figure 1](image-url) Identical allocation of health-care resources for CEA-V and CEA-B when \( V \) is equal to \( 1/\lambda^* \).
for $\lambda^*$, and this is an extremely challenging task. To determine $\lambda^*$, all parameters in the formulation for CEA-B, including the costs and effectiveness of all health-care interventions for all diseases must be specified. Formulating this optimization problem is not possible in practice. Thus, decision makers will not have the necessary information to choose $V$ or $B$ to equate $V$ and $1/\lambda^*$.

Even if policymakers were able to equate $V$ and $1/\lambda^*$ at a given point in time, the two values would almost certainly diverge over time. Two potential changes in the health-care system could cause this divergence: a new intervention may become available, the adoption of which could change the value of $\lambda^*$; the health-care budget ($B$) could change, which again changes the value of $\lambda^*$. For CEA-V and CEA-B to yield identical allocation decisions, the value of $V$ has to be constantly updated to equal $1/\lambda^*$. Since such updating is unlikely to happen in practice, the allocation decisions dictated by CEA-V and CEA-B will almost certainly diverge under real-world conditions.

Choosing the Appropriate CEA Formulation: CEA-V or CEA-B

The relative merits of CEA-V and CEA-B has been argued extensively in the health economic literature [4,11,12]. The CEA-V allocation rule is to adopt therapies whose incremental cost-effectiveness ratios fall below a certain threshold. Gafni and Birch [13] note that decision makers must be aware that use of a constant threshold value per QALY to adopt new health interventions may require an increase in the health-care budget. However, in many cases this may not be feasible, at least in the short term. For instance, national health funds may not obtain increased funding. In this case, these agencies have to make allocation decisions using CEA within a fixed budget, i.e., resort to CEA-B.

The concept that CEA-V places no constraint on health-care expenditure is somewhat misleading. The value $V$ represents an implicit budget constraint, limiting the monetary amount that can be spent to gain one QALY, and arises from the fact that the health-care budget is limited. If the budget were not limited, the value of $V$ would be infinity, and the most effective intervention would always be chosen. Thus, $V$ and $B$ are intrinsically related, and when these two values have the proper relationship, CEA-V and CEA-B lead to identical resource allocation decisions.

A better understanding of the relative merits of the two approaches can be achieved by expanding the optimization problem to include the entire national budget, rather than just the health-care budget. Let $\bar{X}$ represent the amount spent on items other than health care and $B$, the national budget. CEA-B will provide the optimal allocation of health-care resources if the health-care budget is $B = X^* - \bar{X}^*$, where $\bar{X}^*$ is the optimal value of $\bar{X}$. CEA-V will provide the same optimal allocation of health-care resources if $V = 1/\bar{X}^*$, where $\bar{X}^*$ is the shadow price on the societal budget constraint. Even if one starts with the optimal value of $B$, this value can change over time as the national budget and the choice set of interventions change. The same is true of $V$.

From a practical perspective, decision makers may find CEA-V to be a more attractive choice than CEA-B. CEA-B requires the decision maker to formulate an optimization problem with complete knowledge of the cost and effectiveness of all health interventions for all diseases. A new intervention can be evaluated using CEA-V by comparing its incremental cost-effectiveness ratio to $V$, thus avoiding the allocation problem for all diseases while evaluating every new intervention, making CEA-V much easier to apply in practice. Still, decision makers must remember that CEA-V will always require that increases in health-care expenditures be covered by increases in health-care budgets, rather than by reallocation of the available budget as implied by CEA-B. They must also be mindful of the fact that $V$ is a dynamic quantity that varies with budgets and technology.

Discussion

The problem of allocating societal resources among different health-care interventions has been examined using two key methodologies (CEA, CBA).
Considerable debate regarding the differences or equivalence of these two methodologies, has arisen [1,2]. Johannesson [3] argued that CEA and CBA would be equivalent if the WTP per unit of effectiveness is constant for an individual, and this WTP is the same for everyone.

The main reason for the discrepant conclusions of these articles is the definitions of CEA and CBA that are the basis for comparison of these analytical methodologies. Phelps and Mushlin [1] argue that both CEA and CBA definitions, when simplified, reduce to our formulation CEA-V, and conclude that the two methodologies are essentially equivalent. Donaldson [2], on the other hand, uses our definition for CEA (CEA-V) and for CBA (CEA-B), and argues that the two methodologies may provide for different resource allocations. Johannesson uses formulations CEA-V and CEA-B, and then derives the conditions for equivalence of the two. As these authors used different definitions of CEA and CBA, it is not surprising that they arrived at seemingly contradictory conclusions.

The long-standing question of the equivalence of CEA and CBA is addressed in this paper based on formulations of these analytical frameworks as optimization problems. Based on the mathematical programming formulation, CEA (–V) and CBA are equivalent if the expected WTP is the same in every subgroup, and if the WTP per QALY and the QALY gain from an intervention for a random individual in a given subgroup are independent random variables. The second condition requires that an individual’s WTP per QALY have no influence on the QALY gain experienced by that individual from any health intervention. Because our condition for equivalence does not require that the WTP per QALY be equivalent for all individuals, it is less restrictive than the condition provided by Johannesson [3], which is sufficient but unnecessary.

As per Johannesson [3], the conditions for equivalence of CEA and CBA are unlikely to hold in practice, and are likely to yield different resource-allocation decisions. When CEA and CBA do differ in terms of resource allocation, the choice of methodology depends on the theoretical assumptions underlying the analysis. If the analysis is based on principles of welfare economics, CBA is the methodology of choice. Acceptance of the extra-welfarist principles would favor the use of CEA. The distinction between the welfarist and extra-welfarist approaches hinges on how they deal with distribution of wealth and the ability to pay for health interventions. The welfarist approach uses willingness to pay estimates, which may depend on the current distribution of wealth, while the extra-welfarist approach assigns a value to health improvements that is independent of this distribution.

We also examined two different definitions for CEA found in the literature: CEA-V, to choose interventions to maximize societal net benefit based on the value V that society is willing to pay for a unit of effectiveness; and CEA-B, the maximization of health benefits within a given budget. Mathematical programming representations of the two methodologies were used to demonstrate that they can yield identical resource allocations when the monetary value of a unit of benefit in CEA is the reciprocal of the shadow price of budget constraint in CEA-B, as noted by Stinnett and Mullahy [5].

Previous publications on this topic did not explore the issue of multiple optima in detail. We show that CEA-V has multiple optima, only one of these solutions corresponds to the optimal solution for CEA-B. Furthermore in this case, optimal solutions to CEA-V may violate budget constraint of CEA-B.

Although CEA-V and CEA-B can provide the same resource allocation decision when V and B are appropriately related to each other, they are likely to diverge in terms of their resource allocation decisions under dynamic, real-world conditions, where budgets and choice of interventions change. If the two methodologies do in fact yield different allocations, it is difficult to determine which methodology is preferable without examining the problem of allocating all of society’s resources. If the available budget is inflexible, then the decision maker will by default implement the CEA-B solution. However, a discrepancy between CEA-V and CEA-B may be an indication that either the value of V needs updating, or that the decision maker needs additional money for new technology. Thus, the decision maker must be vigilant in updating V and B to keep pace with changes in wealth and technology. Otherwise, both CEA-V and CEA-B frameworks could result in allocation decisions that lower societal welfare.

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References


**Appendix A**

Proposition 1: There exists an optimal solution to CEA-V such that \( N_{ij} = N_i \) for one \( j^* \in J(i) \) and \( N_{ij} = 0 \) for all \( j \neq j^* \), for every subgroup \( i \).

Proof: The simplex method for solving linear programs to is used to prove this proposition. For a formulation with \( n \) variables and \( m \) constraints, a basic feasible solution has \( n-m \) components set to zero, with the rest possibly taking on positive values. Further, at least one optimal solution must be a basic feasible solution [14]. Hence, there exists an optimal solution for CEA-V where at most \( I \) variables are positive. Since we know that the \( N_{ij} \)-s have to sum to \( N_i \) for each subgroup, the only way in which this can be achieved is if \( N_{ij} = N_i \) for one \( j^* \in J(i) \) and \( N_{ij} = 0 \) for all \( j \neq j^* \), for every subgroup \( i \).

**Proposition 2:** The optimal solution to CEA-B may involve treating different individuals in a given subgroup with different interventions.

**Proof:** The proposition can be restated as: it is possible that an optimal solution for CEA-B have \( N_{ij} > 0 \) for more than one intervention \( j \in J(i) \). Based on the simplex algorithm, a basic feasible solution, and hence the optimal solution, can have \( I+1 \) variables that are positive. This would require that \( N_{ij} > 0 \) for more than one intervention \( j \in J(i) \). In fact, there exists an optimal solution to CEA-B where \( N_{ij} > 0 \) for two interventions, \( j_1 \) and \( j_2 \), in one subgroup \( i \) (with \( N_{ij_1} + N_{ij_2} = N_i \)), and \( N_{ij} = N_i \) for one intervention in all other subgroups.

**Proposition 3:** If the optimal solution for CEA-B has \( N_{ij} > 0 \) for two interventions, \( j_1 \) and \( j_2 \) in one subgroup \( i_1 \), then the corresponding CEA-V formulation with \( V = 1/\beta^* \) has an infinite number of alternate optima with a proportion \( \beta \) (0 \( \leq \beta \leq 1 \)) of individuals in subgroup \( i_1 \) receiving intervention \( j_1 \) and proportion \( 1-\beta \) receiving intervention \( j_2 \).

**Proof:** Let \( N_{ij}^* \) be the optimal solution for CEA-V when \( i \neq i_1 \). The optimal objective function value for CEA-V is given by:

\[
\sum_{i \in J} \sum_{j \in J(i)} \{ (V \times E[Q_{ij}]) - E[C_{ij}] \} \times N_{ij}^* + N_i \times \left[ \left( (V \times E[Q_{ij_1}]) - E[C_{ij_1}] \right) \times \beta \right] + \left[ (V \times E[Q_{ij_2}]) - E[C_{ij_2}] \right] \times (1-\beta^*)
\]

We claim that in this is case:

\[
V \times E[Q_{ij_1}] - E[C_{ij_1}] = V \times E[Q_{ij_2}] - E[C_{ij_2}]
\]

If the left hand side of equation \( a2 \) is greater than the right hand side, the optimal objective function value can be improved by setting \( N_{ij_1} = N_i \times \beta^* + \delta \) (\( \delta > 0 \), \( N_i \times \beta^* + \delta < N_i \)), and setting \( N_{ij_2} = N_i - N_{ij_1} \). Similarly if the right hand side is greater than the left hand side, the objective function value can be increased by increasing \( N_{ij_2} \) by \( \delta \), and decreasing \( N_{ij_1} \) by \( \delta \). Hence, the equality in equation \( a2 \) must hold. In this case, for any \( \beta \) (0 \( \leq \beta \leq 1 \)), \( N_{ij} = N_{ij}^* \) for \( i \neq i_1 \), \( N_{ij_1} = N_i \times \beta \), \( N_{ij_2} = 1 - N_{ij_1} \) is an optimal solution to CEA-V.