BUGS Orange Trees: Non-linear growth curve

This dataset was originally presented by Draper and Smith (1981) and reanalysed by Lindstrom and Bates (1990). The data Y_{ij} consist of trunk circumference measurements recorded at time x_j, j=1,...,7 for each of i = 1,..., 5 orange trees. We consider a logistic growth curve as follows:

$$Y_{ij} \sim \text{Normal}(\eta_{ij}, \tau_c)$$

$$\eta_{ij} = \frac{\phi_{i1}}{1 + \phi_{i2} \exp(\phi_{i3} x_j)}$$

$$\theta_{i1} = \log(\phi_{i1})$$
$$\theta_{i2} = \log(\phi_{i2} + 1)$$
$$\theta_{i3} = \log(-\phi_{i3})$$

The BUGS code is as follows

```
model {
    for (i in 1:K) {
        for (j in 1:n) {
            Y[i, j] ~ dnorm(eta[i, j], tauC)
            eta[i, j] <- phi[i, 1] / (1 + phi[i, 2] * exp(phi[i, 3] * x[j]))
        }
        phi[i, 1] <- exp(theta[i, 1])
        phi[i, 2] <- exp(theta[i, 2]) - 1
        phi[i, 3] <- -exp(theta[i, 3])
        for (k in 1:3) {
            theta[i, k] ~ dnorm(mu[k], tau[k])
        }
    }
    tauC ~ dgamma(1.0E-3, 1.0E-3)
    sigmaC <- 1 / sqrt(tauC)
    varC <- 1 / tauC
    for (k in 1:3) {
        mu[k] ~ dnorm(0, 1.0E-4)
        tau[k] ~ dgamma(1.0E-3, 1.0E-3)
        sigma[k] <- 1 / sqrt(tau[k])
    }
}
```

Data

```
⇨list(n = 7, K = 5, x = c(118.00, 484.00, 664.00, 1004.00, 1231.00, 1372.00, 1582.00),
    Y = structure(
        .Data = c(30.00, 58.00, 87.00, 115.00, 120.00, 142.00, 145.00,
                  33.00, 69.00, 111.00, 156.00, 172.00, 203.00, 203.00,
                  30.00, 51.00, 75.00, 108.00, 115.00, 139.00, 140.00,
```

```
                32.00, 62.00, 112.00, 167.00, 179.00, 209.00, 214.00,
                30.00, 49.00, 81.00, 125.00, 142.00, 174.00, 177.00),
        .Dim = c(5, 7)))⇦
```

Inits

```
⇨list(theta = structure(
    .Data = c(5, 2, -6,
              5, 2, -6,
              5, 2, -6,
              5, 2, -6,
              5, 2, -6),
    .Dim = c(5, 3)),
    mu = c(5, 2, -6), tau = c(20, 20, 20), tauC = 20)⇦
```

Results

The Metropolis algorithm is used to sample the theta parameters in this model. The Gaussian proposal distribution used for this algorithm adapts for the first 4000 iterations and these samples are discarded from the summary statistics. A further 1000 update burn-in followed by 10000 updates gave the following parameter estimates:

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
mu[1]	5.263	0.1317	0.005271	5.004	5.262	5.505	5001	10000
mu[2]	2.195	0.1267	0.007693	1.959	2.189	2.443	5001	10000
mu[3]	-5.885	0.1212	0.009366	-6.189	-5.873	-5.695	5001	10000
sigma[1]	0.2257	0.1352	0.004796	0.05804	0.1982	0.5529	5001	10000
sigma[2]	0.1431	0.1267	0.005708	0.02453	0.1086	0.4596	5001	10000
sigma[3]	0.1137	0.1008	0.006555	0.02381	0.08349	0.3696	5001	10000
sigmaC	7.993	1.24	0.04616	6.006	7.85	10.82	5001	10000