# BUGS Inhaler: ordered catagorical data

Ezzet and Whitehead (1993) analyse data from a two-treatment, two-period crossover trial to compare 2 inhalation devices for delivering the drug salbutamol in 286 asthma patients. Patients were asked to rate the clarity of leaflet instructions accompanying each device, using a 4-point ordinal scale. In the table below, the first entry in each cell (r,c) gives the number of subjects in Group 1 (who received device A in period 1 and device B in period 2) giving response r in period 1 and response c in period 2. The entry in brackets is the number of Group 2 subjects (who received the devices in reverse order) giving this response pattern.

| | | *Response in period 2* | | | | |
|---|---|---|---|---|---|---|
| | | **1** **Easy** | **2** **Only clear after re-reading** | **3** **Not very clear** | **4** **Confusing** | **TOTAL** |
| *Response in period 1* | **1** | 59 (63) | 35 (13) | 3 (0) | 2 (0) | 99 (76) |
| | **2** | 11 (40) | 27 (15) | 2 (0) | 1 (0) | 41 (55) |
| | **3** | 0 (7) | 0 (2) | 0 (1) | 0 (0) | 0 (10) |
| | **4** | 1 (2) | 1 (0) | 0 (1) | 0 (0) | 2 (3) |
| | **TOTAL** | 71 (112) | 63 (30) | 5 (2) | 3 (0) | 142 (144) |

The response $R_{it}$ from the *i* th subject (*i* = 1,...,286) in the *t* th period (*t* = 1,2) thus assumes integer values between 1 and 4. It may be expressed in terms of a continuous latent variable $Y_{it}$ taking values on (-inf, inf) as follows:

$$R_{it} = j \quad \text{if} \quad Y_{it} \text{ in } [a_{j-1}, a_j), \quad j = 1,..,4$$

where $a_0$ = -inf and $a4$ = inf. Assuming a logistic distribution with mean $\mu_{it}$ for $Y_{it}$, then the cumulative probability $Q_{itj}$ of subject *i* rating the treatment in period *t* as worse than category *j* (i.e. Prob( $Y_{it}$ >= $a_j$ ) is given by

$$\text{logit}Q_{itj} = -(aj + \mu_{s_i t} + b_i)$$

where $b_i$ represents the random effect for subject *i*. Here, $\mu_{s_i t}$ depends only on the period *t* and the *sequence* $s_i$ = 1,2 to which patient *i* belongs. It is defined as

$$\mu_{11} = \beta / 2 + \pi / 2$$

$$\mu_{12} = -\beta / 2 - \pi / 2 - \kappa$$

$$\mu_{21} = -\beta / 2 + \pi / 2$$

$$\mu_{22} = \beta / 2 - \pi / 2 + \kappa$$

where $\beta$ represents the treatment effect, $\pi$ represents the period effect and $\kappa$ represents the carryover effect. The probability of subject *i* giving response *j* in period *t* is thus given by $p_{itj} = Q_{itj-1} - Q_{itj}$, where $Q_{it0}$ = 1 and

$Q_{it4}$ = 0 (see also the Bones example).

The *BUGS* language for this model is shown below. We assume the $b_i$'s to be normally distributed with zero mean and common precision $\tau$. The fixed effects $\beta$, $\pi$ and $\kappa$ are given vague normal priors, as are the unknown cut points $a_1$, $a_2$ and $a_3$. We also impose order constraints on the latter using the I(,) notation in *BUGS*, to ensure that $a_1 < a_2 < a_3$.

```
model
{
#
# Construct individual response data from contingency table
#
    for (i in 1 : Ncum[1, 1]) {
        group[i] <- 1
        for (t in 1 : T) { response[i, t] <- pattern[1, t] }
    }
    for (i in (Ncum[1,1] + 1) : Ncum[1, 2]) {
        group[i] <- 2 for (t in 1 : T) { response[i, t] <- pattern[1, t] }
    }

    for (k in 2  : Npattern) {
        for(i in (Ncum[k - 1, 2] + 1) : Ncum[k, 1]) {
            group[i] <- 1 for (t in 1 : T) { response[i, t] <- pattern[k, t] }
        }
        for(i in (Ncum[k, 1] + 1) : Ncum[k, 2]) {
            group[i] <- 2 for (t in 1 : T) { response[i, t] <- pattern[k, t] }
        }
    }
#
# Model
#
    for (i in 1 : N) {
        for (t in 1 : T) {
            for (j in 1 : Ncut) {
#
# Cumulative probability of worse response than j
#
                logit(Q[i, t, j]) <- -(a[j] + mu[group[i], t] + b[i])
            }
#
# Probability of response = j
#
            p[i, t, 1] <- 1 - Q[i, t, 1]
            for (j in 2 : Ncut) { p[i, t, j] <- Q[i, t, j - 1] - Q[i, t, j] }
            p[i, t, (Ncut+1)] <- Q[i, t, Ncut]

            response[i, t] ~ dcat(p[i, t, ])
        }
#
# Subject (random) effects
#
        b[i] ~ dnorm(0.0, tau)
}

#
# Fixed effects
#
    for (g in 1 : G) {
```

Note that the data is read into *BUGS* in the original contigency table format to economize on space and effort. The indivdual responses for each of the 286 patients are then constructed within *BUGS*.

## Data ➔ click on one of the arrows to open data 🡐

## Inits ➔ click on one of the arrows to open initail values 🡐

## Results

A 1000 update burn in followed by a further 10000 updates gave the parameter estimates

| node | mean | sd | MC error | 2.5% | median | 97.5% | start | sample |
|---|---|---|---|---|---|---|---|---|
| a[1] | 0.7079 | 0.1377 | 0.004319 | 0.4547 | 0.7022 | 0.9886 | 1001 | 10000 |
| a[2] | 3.91 | 0.3322 | 0.01624 | 3.299 | 3.899 | 4.597 | 1001 | 10000 |
| a[3] | 5.256 | 0.4678 | 0.0186 | 4.394 | 5.237 | 6.22 | 1001 | 10000 |
| beta | 1.047 | 0.3264 | 0.008527 | 0.4203 | 1.039 | 1.707 | 1001 | 10000 |
| kappa | 0.2532 | 0.2524 | 0.006044 | -0.2383 | 0.2547 | 0.7513 | 1001 | 10000 |
| log.sigma | 0.1667 | 0.2279 | 0.01523 | -0.3635 | 0.1984 | 0.5194 | 1001 | 10000 |
| pi | -0.237 | 0.199 | 0.002517 | -0.6342 | -0.2365 | 0.1586 | 1001 | 10000 |
| sigma | 1.21 | 0.2539 | 0.01669 | 0.6953 | 1.219 | 1.681 | 1001 | 10000 |

The estimates can be compared with those of Ezzet and Whitehead, who used the Newton-Raphson method and numerical integration to obtain maximum-likelihood estimates of the parameters. They reported $\beta$ = 1.17 +/- 0.75, $\pi$ = -0.23 +/- 0.20, $\kappa$ = 0.21 +/- 0.49, log$\sigma$ = 0.17 +/- 0.23, a1 = 0.68, a2 = 3.85, a3 = 5.10