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Vignettes for the Year 2000: Theory and Methods
Foreword

George CASELLA

The vignette series concludes with 22 contributions in Theory and Methods. It is, of course, impossible to cover all of theory and methods with so few articles, but we hope that a snapshot of what was, and what may be, is achieved. This is the essence of "vignettes" which, according to the *American Heritage Dictionary*, are "literary sketches having the intimate charm and subtlety attributed to vignette portraits."

Through solicitation and announcement, we have collected the Theory and Methods vignettes presented here. The scope is broad, but by no means exhaustive. Exclusion of a topic does not bear on its importance, but rather on the inability to secure a vignette. Many topics were solicited, and a general call was put in *Amstat News*. Nonetheless, we could not get every topic covered (among other factors, time was very tight).

There is some overlap in the vignettes, as the authors, although aware of the other topics, were working independently, and there are places where information in one vignette complements that in another. Rather than edit out some of the overlap, I have tried to signpost these instances with cross-references allowing the reader the luxury of seeing two (or even three) views on a topic. Such diverse accounts can help to enhance our understanding.

As I hope you will agree, the resulting collection is nothing short of marvelous. The writers are all experts in their fields, and bring a perception and view that truly highlights each subject area. My goal in this introduction is not to summarize what is contained in the following pages, but rather to entice you to spend some time looking through the vignettes. At the very least, you will find some wonderful stories about the history and development of our subject. (For example, see the vignettes by McCulloch and Meng for different histories of the EM algorithm.) Some of the speculation may even inspire you to try your hand, either in developing the theory or applying the methodology.

The question of in which order to present the vignettes was one that I thought hard about. First, I tried to put them in a subject-oriented order, to create some sort of smooth flow throughout. This turned out to be impossible, as the connections between topics is not linear. Moreover, any absolute ordering could carry a connotation of importance of the topics, a judgment that I don't feel qualified to make. (Indeed, such a judgment may be impossible to make.) So in the end I settled for an alphabetical ordering according to author name. This is not only objective, but also makes the various vignettes a bit easier to find.

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Bayesian Analysis: A Look at Today and Thoughts of Tomorrow

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1. INTRODUCTION

Life was simple when I became a Bayesian in the 1970s; it was possible to track virtually all Bayesian activity. Preparing this paper on Bayesian statistics was humbling, as I realized that I have lately been aware of only about 10% of the ongoing activity in Bayesian analysis. One goal of this article is thus to provide an overview of, and access to, a significant portion of this current activity. Necessarily, the overview will be extremely brief; indeed, an entire area of Bayesian activity might only be mentioned in one sentence and with a single reference. Moreover, many areas of activity are ignored altogether, either due to ignorance on my part or because no single reference provides access to the literature.

A second goal is to highlight issues or controversies that may shape the way that Bayesian analysis develops. This material is somewhat self-indulgent and should not be taken too seriously; for instance, if I had been asked to write such an article 10 years ago, I would have missed the mark by not anticipating the extensive development of Markov

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chain Monte Carlo (MCMC) and its enormous impact on Bayesian statistics.

Section 2 provides a brief snapshot of the existing Bayesian activity and emphasizes its dramatic growth in the 1990s, both inside and outside statistics. I found myself simultaneously rejoicing and being disturbed at the level of Bayesian activity. As a Bayesian, I rejoiced to see the extensive utilization of the paradigm, especially among nonstatisticians. As a statistician, I worried that our profession may not be adapting fast enough to this dramatic change; we may be in danger of "losing" Bayesian analysis to other disciplines (as we have "lost" other areas of statistics). In this regard, it is astonishing that most statistics and biostatistics departments in the United States do not even regularly offer a single Bayesian statistics course.

Section 3 is organized by approaches to Bayesian analysis—in particular the objective, subjective, robust, frequentist-Bayes, and what I term quasi-Bayes approaches. This section contains most of my musings about the current and future state of Bayesian statistics. Section 4 briefly discusses the critical issues of computation and software.

2. BAYESIAN ACTIVITY

2.1 Numbers and Organizations

The dramatically increasing level of Bayesian activity can be seen in part through the raw numbers. Harry Martz (personal communication) studied the SciSearch database at Los Alamos National Laboratories to determine the increase in frequency of articles involving Bayesian analysis over the last 25 years. From 1974 to 1994, the trend was linear, with roughly a doubling of articles every 10 years. In the last 5 years, however, there has been a very dramatic upswing in both the number and the rate of increase of Bayesian articles.

This same phenomenon is also visible by looking at the number of books written on Bayesian analysis. During the first 200 years of Bayesian analysis (1769–1969), there were perhaps 15 books written on Bayesian statistics. Over the next 20 years (1970–1989), a guess as to the number of Bayesian books produced is 30. Over the last 10 years (1990–1999), roughly 60 Bayesian books have been written, not counting the many dozens of Bayesian conference proceedings and collections of papers. Bayesian books in particular subject areas are listed in Sections 2.2 and 2.3. A selection of general Bayesian books is given in Appendix A.

Another aspect of Bayesian activity is the diversity of existing organizations that are significantly Bayesian in nature, including the following (those with an active website): International Society of Bayesian Analysis (http://www.bayesian.org), ASA Section on Bayesian Statistical Science (http://www.stat.duke.edu/sbss/sbss.html), Decision Analysis Society of INFORMS (http://www.informs.org/society/da), and ASA Section on Risk Analysis (http://www.isds.duke.edu/riskanalysis/ras.html).

In addition to the activities and meetings of these societies, the following are long-standing series of prominent Bayesian meetings that are not organized explicitly by societies: Valencia Meetings on Bayesian Statistics (http://www.uv.es/~bernardo/valenciam.html), Co ferences on Maximum Entropy and Bayesian Metho (http://omega.albany.edu:8008/maxent.html), CMU Wor shops on Bayesian Case Studies (http://lib.stat.cmu.edu bayesworkshop/), and RSS Conferences on Practic Bayesian Statistics. The average number of Bayesian mee ings per year is now well over 10, with at least an equ number of meetings being held that have a strong Bayesia component.

2.2 Interdisciplinary Activities and Applications

Applications of Bayesian analysis in industry and govern ment are rapidly increasing but hard to document, as the are often "in-house" developments. It is far easier to doc ument the extensive Bayesian activity in other discipline indeed, in many fields of the sciences and engineering, ther are now active groups of Bayesian researchers. Here we ca do little more than list various fields that have seen a cor siderable amount of Bayesian activity, and present a fe references to access the corresponding literature. Most o the listed references are books on Bayesian statistics in th given field, emphasizing that the activity in the field ha reached the level wherein books are being written. Indeec this was the criterion for listing an area, although field in which there is a commensurate amount of activity, bu no book, are also listed. (It would be hard to find an are of human investigation in which there does not exist som level of Bayesian work, so many fields of application ar omitted.)

For **archaeology,** see Buck, Cavanaugh, and Litto (1996); **atmospheric sciences,** see Berliner, Royle, Wikle and Milliff (1999); **economics and econometrics,** see Cy ert and DeGroot (1987), Poirier (1995), Perlman and Blau (1997), Kim, Shephard and Chib (1998), and Gewek (1999); **education,** see Johnson (1997); **epidemiology,** see Greenland (1998); **engineering,** see Godsill and Rayne (1998); **genetics,** see Iversen, Parmigiani, and Berry (1998) Dawid (1999) and Liu, Neuwald, and Lawrence (1999); **hydrology,** see Parent, Hubert, Bobée and Miquel (1998); **law** see DeGroot, Fienberg, and Kadane (1986) and Kadane and Schuan (1996); **measurement and assay,** see Brown (1993) and http://www.pnl.gov/bayesian/; **medicine,** see Berry and Stangl (1996) and Stangl and Berry (1998); **physical sciences,** see Bretthorst (1988), Jaynes (1999), and http://www.astro.cornell.edu/staff/loredo/bayes/; **quality management,** see Moreno and Rios-Insua (1999); **social sciences,** see Pollard (1986) and Johnson and Albert (1999).

2.3 Areas of Bayesian Statistics

Here, Bayesian activity is listed by statistical area. Again, the criterion for inclusion of an area is primarily the amount of Bayesian work being done in that area, as evidenced by books being written (or a corresponding level of papers).

For **biostatistics,** see Berry and Stangl (1996), Carlin and Louis (1996), and Kadane (1996); **causality,** see Spirtes, Glymour, and Scheines (1993) and Glymour and Cooper (1999); **classification, discrimination, neural nets,** and so on, see Neal (1996, 1999), Müller and Rios-Insua (1998), and the vignette by George; **contingency tables,** see the vignette by Fienberg; **decision analysis and decision theory,** see Smith (1988), Robert (1994), Clemen (1996), and the vignette by Brown; **design,** see Pilz (1991), Chaloner and Verdinelli (1995), and Müller (1999); **empirical Bayes,** see Carlin and Louis (1996) and the vignette by Carlin and Louis; **exchangeability and other foundations,** see Good (1983), Regazzini (1999), Kadane, Schervish and Seidenfeld (1999), and the vignette by Robins and Wasserman; **finite-population sampling,** see Bolfarine and Zacks (1992) and Mukhopadhyay (1998); **generalized linear models,** see Dey, Ghosh, and Mallick (2000); **graphical models and Bayesian networks,** see Pearl (1988), Jensen (1986), Lauritzen (1996), Jordan (1998), and Cowell, Dawid, Lauritzen, and Spiegelhalter (1999); **hierarchical (multilevel) modeling,** see the vignette by Hobert; **image processing,** see Fitzgerald, Godsill, Kokaram, and Stark (1999); **information,** see Barron, Rissanen, and Yu (1998) and the vignette by Soofi; **missing data,** see Rubin (1987) and the vignette by Meng; **nonparametrics and function estimation,** see Dey, Müller, and Sinha (1998), Müller and Vidakovic (1999), and the vignette by Robins and Wasserman; **ordinal data,** see Johnson and Albert (1999); **predictive inference and model averaging,** see Aitchison and Dunsmore (1975), Leamer (1978), Geisser (1993), Draper (1995), Clyde (1999), and the BMA website under "software"; **reliability and survival analysis,** see Clarotti, Barlow, and Spizzichino (1993) and Sinha and Dey (1999); **sequential analysis,** see Carlin, Kadane, and Gelfand (1998) and Qian and Brown (1999); **signal processing,** see Ó Ruanaidh and Fitzgerald (1996) and Fitzgerald, Godsill, Kokaram, and Stark (1999); **spatial statistics,** see Wolpert and Ickstadt (1998) and Besag and Higdon (1999); **testing, model selection, and variable selection,** see Kass and Raftery (1995), O'Hagan (1995), Berger and Pericchi (1996), Berger (1998), Racugno (1998), Sellke, Bayarri, and Berger (1999), Thiesson, Meek, Chickering, and Heckerman (1999), and the vignette by George; **time series,** see Pole, West, and Harrison (1995), Kitagawa and Gersch (1996) and West and Harrison (1997).

3. APPROACHES TO BAYESIAN ANALYSIS

This section presents a rather personal view of the status and future of five approaches to Bayesian analysis, termed the objective, subjective, robust, frequentist-Bayes, and quasi-Bayes approaches. This is neither a complete list of the approaches to Bayesian analysis nor a broad discussion of the considered approaches. The section's main purpose is to emphasize the variety of different and viable Bayesian approaches to statistics, each of which can be of great value in certain situations and for certain users. We should be aware of the strengths and weaknesses of each approach, as all will be with us in the future and should be respected as part of the Bayesian paradigm.

3.1 Objective Bayesian Analysis

It is a common perception that Bayesian analysis is primarily a subjective theory. This is true neither historically nor in practice. The first Bayesians, Thomas Bayes (see Bayes 1783) and Laplace (see Laplace 1812), performed Bayesian analysis using a constant prior distribution for unknown parameters. Indeed, this approach to statistics, then called "inverse probability" (see Dale 1991) was very prominent for most of the nineteenth century and was highly influential in the early part of this century. Criticisms of the use of a constant prior distribution caused Jeffreys to introduce significant refinements of this theory (see Jeffreys 1961). Most of the applied Bayesian analyses I see today follow the Laplace–Jeffreys objective school of Bayesian analysis, possibly with additional modern refinements. (Of course, others may see subjective Bayesian applications more often, depending on the area in which they work.)

Many Bayesians object to the label "objective Bayes," claiming that it is misleading to say that any statistical analysis can be truly objective. Though agreeing with this at a philosophical level (Berger and Berry 1988), I feel that there are a host of practical and sociological reasons to use the label; statisticians must get over their aversion to calling good things by attractive names.

The most familiar element of the objective Bayesian school is the use of *noninformative* or *default* prior distributions. The most famous of these is the *Jeffreys prior* (see Jeffreys 1961). *Maximum entropy* priors are another well-known type of noninformative prior (although they often also reflect certain informative features of the system being analyzed). The more recent statistical literature emphasizes what are called *reference priors* (Bernardo 1979; Yang and Berger 1997), which prove remarkably successful from both Bayesian and non-Bayesian perspectives. Kass and Wasserman (1996) provided a recent review of methods for selecting noninformative priors.

A quite different area of the objective Bayesian school is that concerned with techniques for default model selection and hypothesis testing. Successful developments in this direction are much more recent (Berger and Pericchi 1996; Kass and Raftery 1995; O'Hagan 1995; Sellke, Bayarri, and Berger 1999). Indeed, there is still considerable ongoing discussion as to which default methods are to be preferred for these problems (see Racugno 1998).

The main concern with objective Bayesian procedures is that they often utilize improper prior distributions, and so do not automatically have desirable Bayesian properties, such as coherency. Also, a poor choice of improper priors can even lead to improper posteriors. Thus proposed objective Bayesian procedures are typically studied to ensure that such problems do not arise. Also, objective Bayesian procedures are often evaluated from non-Bayesian perspectives, and usually turn out to be stunningly effective from these perspectives.

3.2 Subjective Bayesian Analysis

Although comparatively new on the Bayesian scene, subjective Bayesian analysis is currently viewed by many Bayesians to be the "soul" of Bayesian statistics. Its philosophical appeal is undeniable, and few statisticians would argue against its use when the needed inputs (models and

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subjective prior distributions) can be fully and accurately specified. The difficulty in such specification (Kahneman, Slovic, and Tversky 1986) often limits application of the approach, but there has been a considerable research effort to further develop elicitation techniques for subjective Bayesian analysis (Lad, 1996; French and Smith 1997; *The Statistician,* 47, 1998).

In many problems, use of subjective prior information is clearly essential, and in others it is readily available; use of subjective Bayesian analysis for such problems can provide dramatic gains. Even when a complete subjective analysis is not feasible, judicious use of partly subjective and partly objective prior distributions is often attractive (Andrews, Berger, and Smith 1993).

3.3 Robust Bayesian Analysis

Robust Bayesian analysis recognizes the impossibility of complete subjective specification of the model and prior distribution; after all, complete specification would involve an infinite number of assessments, even in the simplest situations. The idea is thus to work with classes of models and classes of prior distributions, with the classes reflecting the uncertainty remaining after the (finite) elicitation efforts. (Classes could also reflect the differing judgments of various individuals involved in the decision process.)

The foundational arguments for robust Bayesian analysis are compelling (Kadane 1984; Walley 1991), and there is an extensive literature on the development of robust Bayesian methodology, including Berger (1985, 1994), Berger et al. (1996), and Rios Insua (1990). Routine practical implementation of robust Bayesian analysis will require development of appropriate software, however.

Robust Bayesian analysis is also an attractive technology for actually implementing a general subjective Bayesian elicitation program. Resources (time and money) for subjective elicitation typically are very limited in practice, and need to be optimally utilized. Robust Bayesian analysis can, in principle, be used to direct the elicitation effort, by first assessing if the current information (elicitations and data) is sufficient for solving the problem and then, if not, determining which additional elicitations would be most valuable (Liseo, Petrella, and Salinetti 1996).

3.4 Frequentist Bayes Analysis

It is hard to imagine that the current situation, with several competing foundations for statistics, will exist indefinitely. Assuming that a unified foundation is inevitable, what will it be? Today, an increasing number of statisticians envisage that this unified foundation will be a mix of Bayesian and frequentist ideas (with elements of the current likelihood theory thrown in; see the vignette by Reid). Here is my view of what this mixture will be.

First, the language of statistics will be Bayesian. Statistics is about measuring uncertainty, and over 50 years of efforts to prove otherwise have convincingly demonstrated that the only coherent language in which to discuss uncertainty is the Bayesian language. In addition, the Bayesian language is an order of magnitude easier to understand than the classical language (witness the p value controversy Sellke et al. 1999), so that a switch to the Bayesian language should considerably increase the attractiveness of statistics Note that, as discussed earlier, this is not about subjectivity or objectivity; the Bayesian language can be used for either subjective or objective statistical analysis.

On the other hand, from a methodological perspective, it is becoming clear that both Bayesian and frequentist methodology is going to be important. For parametric problems, Bayesian analysis seems to have a clear methodological edge, but frequentist concepts can be very useful, especially in determining good objective Bayesian procedures (see, e.g., the vignette by Reid).

In nonparametric analysis, it has long been known (Diaconis and Freedman 1986) that Bayesian procedures can behave poorly from a frequentist perspective. Although poor frequentist performance is not necessarily damning to a Bayesian, it typically should be viewed as a warning sign that something is amiss, especially when the prior distribution used contains more "hidden" information than elicited information (as is virtually always the case with nonparametric priors).

Furthermore, there are an increasing number of examples in which frequentist arguments yield satisfactory answers quite directly, whereas Bayesian analysis requires a formidable amount of extra work. (The simplest such example is MCMC itself, in which one evaluates an integral by a sample average, and not by a formal Bayesian estimate; see the vignette by Robins and Wasserman for other examples). In such cases, I believe that the frequentist answer can be accepted by Bayesians as an approximate Bayesian answer, although it is not clear in general how this can be formally verified.

This discussion of unification has been primarily from a Bayesian perspective. From a frequentist perspective, unification also seems inevitable. It has long been known that "optimal" unconditional frequentist procedures must be Bayesian (Berger 1985), and there is growing evidence that this must be so even from a conditional frequentist perspective (Berger, Boukai, and Wang 1997).

Note that I am *not* arguing for an eclectic attitude toward statistics here; indeed, I think the general refusal in our field to strive for a unified perspective has been the single biggest impediment to its advancement. I am simply saying that any unification that will be achieved will almost necessarily have frequentist components to it.

3.5 Quasi-Bayesian Analysis

There is another type of Bayesian analysis that one increasingly sees being performed, and that can be unsettling to "pure" Bayesians and many non-Bayesians. In this type of analysis, priors are chosen in various ad hoc fashions, including choosing vague proper priors, choosing priors to "span" the range of the likelihood, and choosing priors with tuning parameters that are adjusted until the answer "looks nice." I call such analyses *quasi-Bayes* because, although they utilize Bayesian machinery, they do not carry the guarantees of good performance that come with either true subjective analysis or (well-studied) objective Bayesian analysis. It is useful to briefly discuss the possible problems with each of these quasi-Bayes procedures.

Using vague proper priors will work well when the vague proper prior is a good approximation to a good objective prior, but this can fail to be the case. For instance, in normal hierarchical models with a "higher-level" variance V, it is quite common to use the vague proper prior density $\pi(V) \propto V^{-(\varepsilon+1)} \exp(-\varepsilon'/V)$, with ε and ε' small. However, as $\varepsilon \to 0$, it is typically the case in these models that the posterior distribution for V will pile up its mass near 0, so that the answer can be ridiculous if ε is too small. An objective Bayesian who incorrectly used the related prior $\pi(V) \propto V^{-1}$ would typically become aware of the problem, because the posterior would not converge (as it will with the vague proper prior). The common perception that using a vague proper prior is safer than using improper priors, or conveys some type of guarantee of good performance, is simply wrong.

The second common quasi-Bayes procedure is to choose priors that span the range of the likelihood function. For instance, one might choose a uniform prior over a range that includes most of the "mass" of the likelihood function but that does not extend too far (thus hopefully avoiding the problem of using a "too vague" proper prior). Another version of this procedure is to use conjugate priors, with parameters chosen so that the prior is considerably more spread out than the likelihood function but is roughly centered in the same region. The two obvious concerns with these strategies are that (a) the answer can still be quite sensitive to the spread of the rather arbitrarily chosen prior, and (b) centering the prior on the likelihood is a problematical double use of the data. Also, in problems with complicated likelihoods, it can be difficult to implement this strategy successfully.

The third common quasi-Bayes procedure is to write down proper (often conjugate) priors with unspecified parameters, and then treat these parameters as "tuning" parameters to be adjusted until the answer "looks nice." Unfortunately, one is sometimes not told that this has been done; that is, the choice of the parameters is, after the fact, presented as "natural."

These issues are complicated by the fact that in the hands of an expert Bayesian analyst, the quasi-Bayes procedures mentioned here can be quite reasonable, in that the expert may have the experience and skill to tell when the procedures are likely to be successful. Also, one must always consider the question: What is the alternative? I have seen many examples in which an answer was required and in which I would trust the quasi-Bayes answer more than the answer from any feasible alternative analysis.

Finally, it is important to recognize that the genie cannot be put back into the bottle. The Bayesian "machine," together with MCMC, is arguably the most powerful mechanism ever created for processing data and knowledge. The quasi-Bayes approach can rather easily create procedures of astonishing flexibility for data analysis, and its use to create such procedures should not be discouraged. However, it must be recognized that these procedures do not necessarily have intrinsic Bayesian justifications, and so must be justified on extrinsic grounds (e.g., through extensive sensitivity studies, simulations, etc.).

4. COMPUTATION AND SOFTWARE

4.1 Computational Techniques

Even 20 years ago, one often heard the refrain that "Bayesian analysis is nice conceptually; too bad it is not possible to compute Bayesian answers in realistic situations." Today, truly complex models often can only be computationally handled by Bayesian techniques. This has attracted many newcomers to the Bayesian approach and has had the interesting effect of considerably reducing discussion of "philosophical" arguments for and against the Bayesian position.

Although other goals are possible, most Bayesian computation is focused on calculation of posterior expectations, which are typically integrals of one to thousands of dimensions. Another common type of Bayesian computation is calculation of the posterior mode (as in computating MAP estimates in image processing).

The traditional numerical methods for computing posterior expectations are numerical integration, Laplace approximation, and Monte Carlo importance sampling. Numerical integration can be effective in moderate (say, up to 10) dimensional problems. Modern developments in this direction were discussed by Monahan and Genz (1996). Laplace and other saddlepoint approximations are discussed in the vignette by R. Strawderman. Until recently, Monte Carlo importance sampling was the most commonly used traditional method of computing posterior expectations. The method can work in very large dimensions and has the nice feature of producing reliable measures of the accuracy of the computation.

Today, MCMC has become the most popular method of Bayesian computation, in part because of its power in handling very complex situations and in part because it is comparatively easy to program. Because the Gibbs sampling vignette by Gelfand and the MCMC vignette by Cappé and Robert both address this computational technique, I do not discuss it here. Recent books in the area include those of Chen, Shao, and Ibrahim (2000), Gamerman (1997), Robert and Casella (1999), and Tanner (1993). It is not strictly the case that MCMC is replacing the more traditional methods listed above. For instance, in some problems importance sampling will probably always remain the computational method of choice, as will standard numerical integration in low dimensions (especially when extreme accuracy is needed).

Availability of general user-friendly Bayesian software is clearly needed to advance the use of Bayesian methods. A number of software packages exist, and these are very useful for particular scenarios. Lists and description of pre-1990 Bayesian software were provided by Goel (1988) and Press (1989). A list of some of the Bayesian software developed since 1990 is given in Appendix B.

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It would, of course, be wonderful to have a single general-purpose Bayesian software package, but three of the major strengths of the modern Bayesian approach create difficulties in developing generic software. One difficulty is the extreme flexibility of Bayesian analysis, with virtually any constructed model being amenable to analysis. Most classical packages need to contend with only a relatively few well-defined models or scenarios for which a classical procedure has been determined. Another strength of Bayesian analysis is the possibility of extensive utilization of subjective prior information, and many Bayesians tend to feel that software should include an elaborate expert system for prior elicitation. Finally, implementing the modern computational techniques in a software package is extremely challenging, because it is difficult to codify the "art" of finding a successful computational strategy in a complex situation.

Note that development of software implementing the objective Bayesian approach for "standard" statistical models can avoid these difficulties. There would be no need for a subjective elicitation interface, and the package could incorporate specific computational techniques suited to the various standard models being considered. Because the vast majority of statistical analyses done today use such "automatic" software, having a Bayesian version would greatly impact the actual use of Bayesian methodology. Its creation should thus be a high priority for the profession.

APPENDIX A: GENERAL BAYESIAN REFERENCES

- Historical and general monographs: Laplace (1812), Jeffreys (1961), Zellner (1971), Savage (1972), Lindley (1972), Box and Tiao (1973), de Finetti (1974, 1975), Hartigan (1983), Florens, Mouchart, and Roulin (1990)
- Graduate-level texts: DeGroot (1970), Berger (1985), Press (1989), Bernardo and Smith (1994), O'Hagan (1994), Robert (1994), Gelman, Carlin, Stern, and Rubin (1995), Poirier (1995), Schervish (1995), Piccinato (1996)
- Elementary texts: Winkler (1972), O'Hagan (1988), Albert (1996), Berry (1996), Sivia (1996), Antleman (1997), Lee (1997)
- General proceedings volumes: The International Valencia Conferences produce highly acclaimed proceedings, the last of which was edited by Bernardo et al. (1999). The Maximum Entropy and Bayesian Analysis conferences also have excellent proceedings volumes, the last of which was edited by Erickson, Rychert, and Smith (1998). The CMU Bayesian Case Studies Workshops produce unique volumes of in-depth case studies in Bayesian analysis, the last volume being edited by Gatsonis et al. (1998). The Bayesian Statistical Science Section of the ASA has an annual JSM proceedings volume, produced by the ASA.

APPENDIX B: AVAILABLE BAYESIAN SOFTWARE

- AutoClass, a Bayesian classification system (http://ic-www.arc.nasa.gov/ic/projects/bayes-group/group/autoclass/)
- BATS, designed for Bayesian time series analysis (http://www.stat.duke.edu/~mw/bats.html)
- BAYDA, a Bayesian system for classification and discriminant analysis (http://www.cs.Helsinki.fi/research/cosco/Projects/NONE/SW/)
- BAYESPACK, etc., numerical integration algorithms (http:/www.math.wsu.edu/math/faculty/genz/homepage)
- Bayesian biopolymer sequencing software (http://www stat.stanford.edu/~jliu/)
- B/D, a linear subjective Bayesian system (http://fourie dur.ac.uk:8000/stats/bd/)
- BMA, software for Bayesian model averaging for predictiv and other purposes (http://www.research.att.com/~volinsk /bma.html)
- Bayesian regression and classification software based o neural networks, Gaussian processes, and Bayesian mixtur models (http://www.cs.utoronto.ca/~radford/fbm.softwar html)
- Belief networks software (http://bayes.stat.washington.edu almond/belief.html)
- BRCAPRO, which implements a Bayesian analysis for genetic counseling of women at high risk for hereditar breast and ovarian cancer (http://www.stat.duke.edu/~gp, brcapro.html)
- BUGS, designed to analyze general hierarchical models vi MCMC (http://www.mrc-bsu.cam.ac.uk/bugs/)
- First Bayes, a Bayesian teaching package (http://www.shef ac.uk/~st1ao/1b.html)
- Matlab and Minitab Bayesian computational algorithm for introductory Bayes and ordinal data (http://www math.bgsu.edu/~albert/)
- Nuclear magnetic resonance Bayesian software; this is th manual (http://www.bayes.wustl.edu/glb/manual.pdf)
- StatLib, a repository for statistics software, much of i Bayesian (http://lib.stat.cmu.edu/)
- Time series software for nonstationary time serie and analysis with autoregressive component model (http://www.stat.duke.edu/~mw/books_software_data.html
- LISP-STAT, an object-oriented environment for statistica computing and dynamic graphics with various Bayesian capabilities (Tierney 1991)

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