Bayesian Analysis in Medicine EPIB-677

Points for Discussion

1. Precisely what information does a p-value provide?
2. What is the correct (and incorrect) way to interpret a confidence interval?
3. What is Bayes Theorem? How does it operate?
4. Summary of above three points: What exactly does one learn about a parameter of interest after having done a frequentist analysis? Bayesian analysis?
5. Example 8 from the book chapter by Jim Berger.
6. Example 13 from the book chapter by Jim Berger.
7. Example 17 from the book chapter by Jim Berger.
8. Examples where there is a choice between a binomial or a negative binomial likelihood, found in the paper by Berger and Berry.
9. Problems with specifying prior distributions.
10. In what types of epidemiology data analysis situations are Bayesian methods particularly useful?
11. What does decision analysis add to a Bayesian analysis?

1. **Precisely what information does a p-value provide?**

   Recall the definition of a p-value: The probability of observing a test statistic as extreme as or more extreme than the observed value, assuming that the null hypothesis is true.

2. **What is the correct (and incorrect) way to interpret a confidence interval?**

   Does a 95% confidence interval provide a 95% probability region for the true parameter value? If not, what it the correct interpretation?

   In practice, it is usually helpful to consider the following graphic:

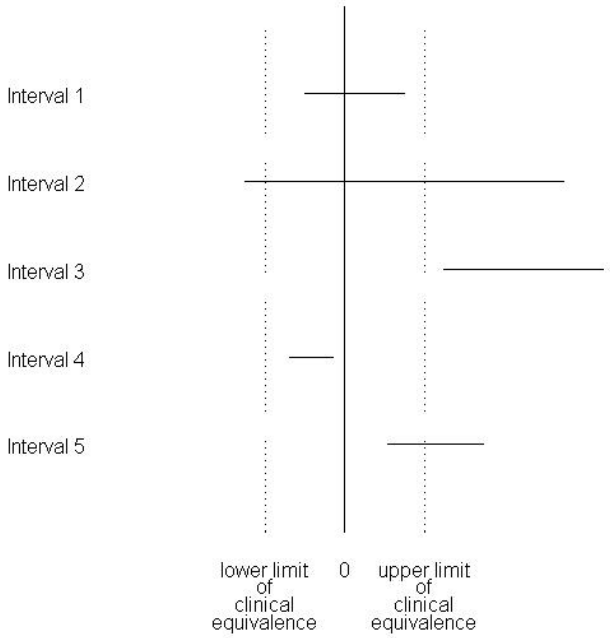

Figure 1: How to interpret confidence intervals and/or credible regions. Depending on where the confidence/credible interval lies in relation to a region of clinical equivalence, different conclusions can be drawn.

Depending on where the upper and lower confidence/credible interval limits fall in relation to the upper and lower limits of the region of clinical equivalence, different conclusions should be drawn. The region of clinical equivalence, sometimes called the region of clinical indifference, is the region inside of which two treatments, say, would be considered to be the same for all practical purposes. The point 0, indicating no difference in results between two treatments, is usually included in the region of clinical equivalence, but values above and below 0 are usually also included. How wide this region is depends on each individual clinical situation. For example, if one treatment is much more expensive than another, one may want at least a 5% advantage in order to consider it the preferred treatment (see Chapter 15). From the figure, there are five different conclusions that can be made after a confidence or credible interval has been calculated:

(a) The CI includes zero, and both upper and lower CI limits, if they were the true values, would not be clinically interesting. Therefore, this variable has been shown to have no important effect.

(b) The CI includes zero, but one or both of the upper or lower CI limits, if they were the true values, would be interesting clinically. Therefore, the results of this variable in this study is inconclusive, and further evidence needs to be collected.

(c) The CI does not include zero, and all values inside the upper and lower CI limits, if they were the true values, would be clinically interesting. Therefore, this study shows this variable to be important.

(d) The CI does not include zero, but all values inside the upper and lower CI limits, if they were the true values, would not be clinically interesting. Therefore, this study shows this variable, while having some small effect, is not clinically important.

(e) The CI does not include zero, but only some of the values inside the upper and lower CI limits, if they were the true values, would be clinically interesting. Therefore, this study shows this variable

has at least a small effect, and may be clinically important. Further study is required in order to better estimate the magnitude of this effect.

3. **What is Bayes Theorem? How does it operate?**

   Continuous version:

$$f(\theta|x) = \frac{f(x|\theta) \times f(\theta)}{\int f(x|\theta) \times f(\theta) d\theta,}$$

   Discrete Version:

$$P(B_k|A) = \frac{P(B_k) \times P(A|B_i)}{\sum_{i=1}^{n} P(B_i) \times P(A|B_i)},\ k = 1, 2, \ldots, n.$$

   What information does one have after a Bayesian analysis is carried out?

4. **Summary of above three points: What exactly does one learn about a parameter of interest after having done a frequentist analysis? Bayesian analysis?**

   Suppose you have some data that need to be analyzed. Suppose further there are two competing firms, Bayes 'R Us, and Freq Inc.

   What information would you have to give to Freq Inc., in order for them to provide you with an analysis? What would they give you in return? What would the (relative) cost be?

   What information would you have to give to Bayes 'R Us, in order for them to provide you with an analysis? What would they give you in return? What would the (relative) cost be?

   What is the difference between a p-value and the posterior probability of a null hypothesis $Pr\{H_0 \mid \text{the data}\}$.

5. **Example 8 from the book by Jim Berger.**

6. **Example 13 from the book by Jim Berger.**

7. **Example 17 from the book by Jim Berger.**

8. **Examples where there is a choice between a binomial or a negative binomial likelihood, found in the paper by Berger and Berry.**

   Does it make intuitive sense for there to be a difference between using a binomial or a negative binomial model for the same data? Why or why not?

   Discuss the phrase (page 163) "...science earns its reputation for objectivity by treating the perils of subjectivity with the greatest respect."

9. **Problems with specifying prior distributions.**

   Where does the prior come from?

   What happens if different investigators have different prior distributions?

   Is this an advantage or a disadvantage of the Bayesian approach?

10. **In what types of epidemiology data analysis situations are Bayesian methods particularly useful?**

    See the examples discussed in the article by Dunson.

11. **What does decision analysis add to a Bayesian analysis?**

    Remember the basic elements.