Principles of Inferential Statistics in Medicine

Midterm Exam – 513–607A, October 24th, 2000.

1. A large population based cross-sectional study is carried out to investigate the association between daily calcium intake (measured in mg) and osteoporosis in women over the age of 65. A glass of milk contains about 300 mg of calcium, and the recommended daily intake is 1200 mg. The investigators divide the sample into those who are osteoporotic and those without this disease, and calculate the average calcium intake in each group. The results are as follows:

subjects with osteoporosis	subjects without osteoporosis
$\bar{x}_1 = 820$	$\bar{x}_2 = 800$
$s_1 = 400$	$s_2 = 400$
$n_1 = 3500$	$n_2 = 3500$

Test the null hypothesis that average daily calcium intakes in subjects with and without osteoporosis are the same, versus a two-sided alternative. State the null and alternative hypotheses, carry out the test, provide a p-value, and state your conclusion.

2. A clinical trial was conducted to determine the effect of β-blocker drugs on the pulse rate (given in number of pulses per minute) of patients during surgery. The results are given below:

beta-blocker group	placebo group
$\bar{x}_1 = 65$	$\bar{x}_2 = 67$
$s_1 = 20$	$s_2 = 12$
$n_1 = 16$	$n_2 = 16$

(a) Calculate a 95% confidence interval for the difference in pulse rates in the two groups (placebo group - beta-blocker group).

(b) Based on your answer in (a), provide a conclusion about the effects of beta-blockers on pulse rates during surgery.

3. In a pilot study measuring the average blood pressure of men aged 50 to 55, the blood pressures of three men in this age group are measured. Their diastolic pressures are found to be 70, 80, and 90 $mm\ Hg$. The investigators of the study state that their prior distribution for the mean blood pressure among such subjects is normally distributed, with mean 75 and variance 25. They also claim to exactly know the standard deviation of blood pressures among men aged 50 to 55 to be 10 $mm\ Hg$.

What is the posterior distribution for the mean blood pressure in this age group, taking into account the prior distribution and the data collected? Provide the mean and variance of this posterior distribution, as well as a rough sketch of this distribution.

4.   Assume that X, Y, and Z are all independent random variables, each having a N($\mu = 0$, $\sigma^2 = 1$) distribution.

(a) What is the distribution of $(5 \times X) - (3 \times Y)$?

(b) What is the distribution of $\frac{X+Y+Z}{3}$?

5.   Calculate the following probabilities:

(a) If X follows a Poisson distribution with mean $\mu = 2$, what is $\Pr\{X \geq 2\}$?

(b) If Y follows a binomial distribution with probability of "success" equal to 0.3 and with 400 trials, what is $\Pr\{Y = 120\}$?

6.   A diagnostic test for the diagnosis of disease "A" in a given population has positive predictive value of 60%. If the sensitivity of the test is 90% and the specificity is 80%, what is the prevalence of disease "A" in this population?