Session 5: Outline
Chapter 11: Interaction(Effect Modification) in Regression

Preamble
 - Should not be in same chapter with confounding...
 - a very different topic !! (can have both, but ... see diagram)

Definitions ...

Interaction (statistical)

- "Non-additivity" of "effects" in regression

- need for product term in regression analysis (osm)

- scale dependent

(Effect) Modification (epidemiological)

- Inconstancy of a parameter of a relation over other subject characteristic (osm)

- Different slopes for different folks (jh)

"Modifier (of a relation)
- A characteristic (of individuals) on which a parameter of a relation depends (osm)

Examples...

Equation for Ideal Weight as function of Height

- *modification by Gender*

Average Earnings as function of Education / Age

- *modification by Gender*

Decline in Bone Density with Age

- *Different in 19th and 20th Centuries*

?Can hit further with aluminum than wood baseball bat?

- *Difference depends on where on bat one hits ball*

Changes over time in injury rates

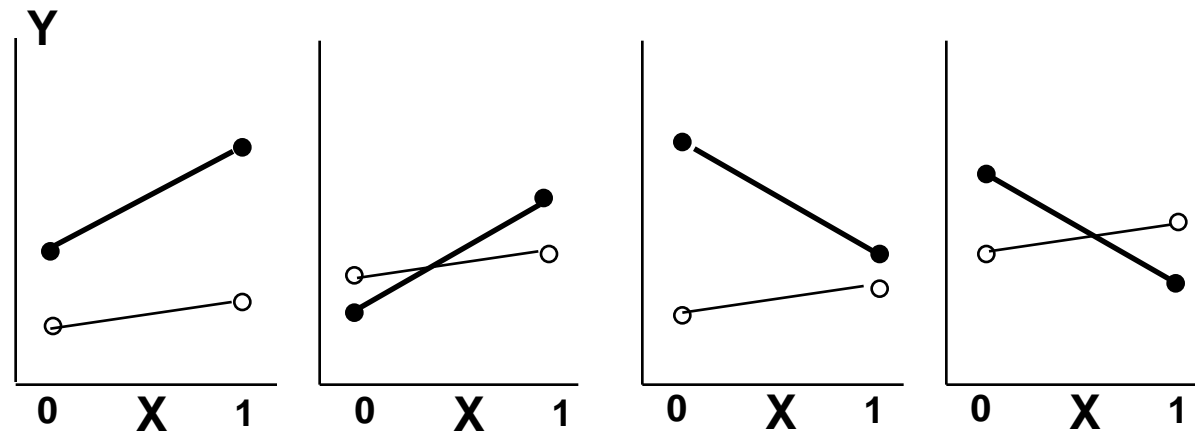- *Different in intervention and reference areas?*

Translating these into regression equations ...
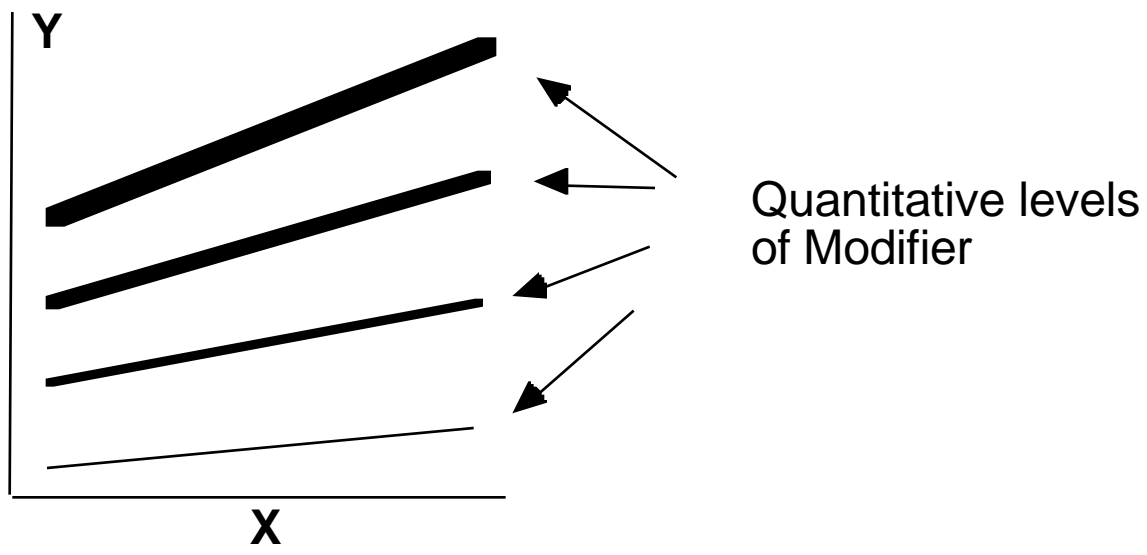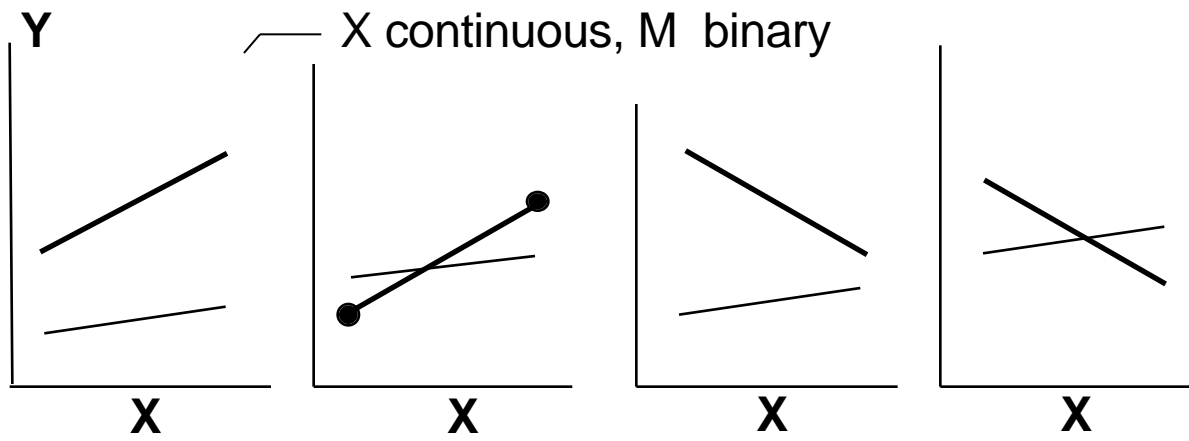
- relation between Y and X

- "modifier" variable M

    E[Y | X, M] = B0 + B1.X + B2.M + B3.(M.X)
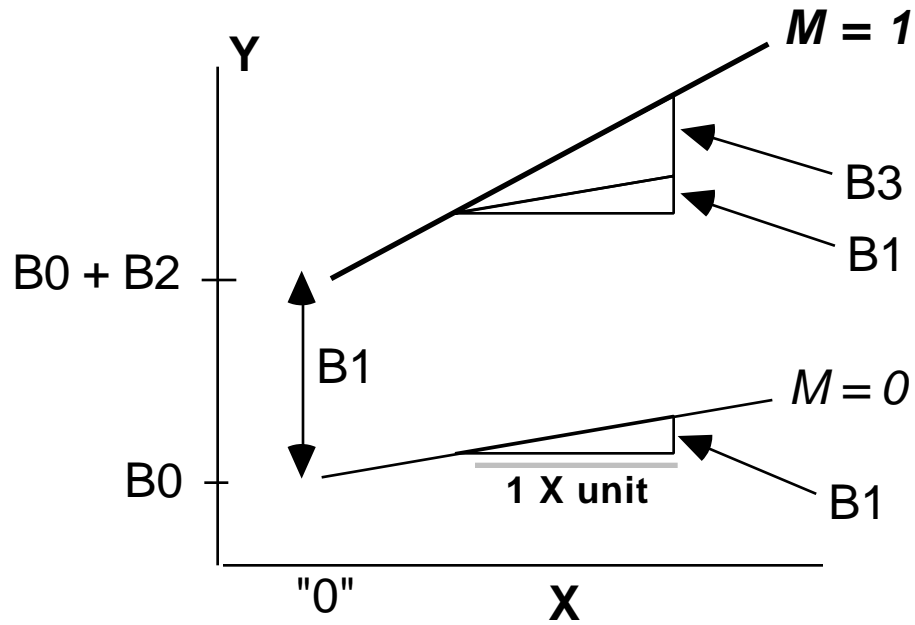
- Special cases..

    X binary, M Binary

━━━━━ **Modifier = 1**

───── Modifier = 0

Y   X continuous, M binary

X   **X**   **X**   **X**

Y

Quantitative levels
of Modifier

X

Meaning of the coefficients

      X continuous, M Binary

  - helpful ways of rewriting the equation

     E[Y | X, M] = B0 + B2.M + (B1 + B3.M).X

Special issues

- mathematical symmetry of equation

$$E[Y \mid X1, X2] = B0 + B1.X1 + B2.X2 + B3.(X1.X2)$$

$$= B0 + B2.X2 + (B1 + B3.X2).X1$$

X2 modifies the Y<->X1 relation

$$= B0 + B1.X1 + (B2 + B3.X1).X2$$

X1 modifies the Y<->X2 relation

- to a regression program, X1.X2 product terms are just like any other terms.. but

  they tend to be correlated (collinear) with the components from which they are made, so...

  *** user should "center" the components before ***
  *** making (or having computer make) products ***

  (will see example in injury prevention study)

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Translating equations back into lines ...

- <u>If M is binary</u>...

    start with the M=0 case

$$B0 + B1.X + B2.M + B3.(M.X)$$
$$= B0 + B1.X + B2.0 + B3.(0.X)$$

$$= B0 + B1.X$$
===> straight line in X with intercept B0 and slope B1

    "turn on" the M=1 toggle...

$$B0 + B1.X + B2.M + B3.(M.X)$$
$$= B0 + B1.X + B2.1 + B3.(1.X)$$

$$= B0 + B1.X + B2 + B3.X$$

   collect terms that do not involve X & those that do..

$$(B0 + B2) + (B1 + B3).X$$
===> straight line in X with intercept (B0 + B2) and slope (B1 + B3)

- <u>If M is continuous</u>... as above with several M values