(from Feller) **OBSERVATIONS FITTING THE POISSON DISTRIBUTION**[12]

***(a) Radioactive disintegrations*.** A radioactive substance emits alpha-particles; the number of particles reaching a given portion of space during time *t is* the best-known example of random events obeying the Poisson law. Of course, the substance continues to decay, and in the long run the density of alpha-particles will decline. However, with radium it takes years before a decrease of matter can be detected; for relatively short periods the conditions may be considered constant, and we have an ideal realization of the hypotheses which led to the Poisson distribution.

In a famous experiment (Rutherford, Chadwick, and Elllis *Radiations from radioactive substances,* Cambridge 1920, p.172.) a radioactive substance was observed during N = 2608 time intervals of 7.5 seconds each; the number of particles reaching a counter was obtained for each period. Table 3 records the number N_k of periods with exactly k particles. The total number of particles is T= $\Sigma\, k\, N_k$ = 10,094, the average T/N = 3.870. The theoretical values $N_{Poisson}(k;$ 3.870) are seen to be rather close to the observed numbers N_k. To judge the closeness of fit, an estimate of the probable magnitude of chance fluctuations is required. Statisticians judge the closeness of fit by the X^2-criterion. Measuring by this standard, we should expect that under ideal conditions about 17 out of 100 comparable cases would show worse agreement than exhibited in table 3.

TABLE 3 EXAMPLE *(a):* RADIOACTIVE DISINTEGRATIONS

k	N_k	$N_{Poisson}(k;$ 3.870)	k	N_k	$N_{Poisson}(k;$ 3.870)
O	57	54.4	5	408	393.5
1	203	210.5	6	273	253.8
2	383	407.4	7	139	140.3
3	525	525.4	8	45	67.9
4	532	508.4	9	27	29.2
			$k \geq 10$	16	17.1
			Total	2608	2608.0

12 The Poisson distribution has become known as the law of small numbers or of rare events. These are misnomers which proved detrimental to the realization of the fundamental role of the Poisson distribution. The following examples will show how misleading the two names are.

(b) Flying-bomb hits on London. As an example of a spatial distribution of random points consider the statistics of flying-bomb hits in the south of London during World War II. The entire area is divided into $N = 576$ small areas of $t = 1/4$ square kilometers each, and table 4 records the number *N.* of areas with exactly *k* hits.[14] The total number of hits is $T = \Sigma k N_k = 537$, the average $\lambda t = T/N = 0.9323$ .... The fit of the Poisson distribution is surprisingly good; as judged by the X^2-criterion, under ideal conditions some 88 per cent of comparable observations should show a worse agreement. It is interesting to note that most people believed in a tendency of the points of impact to cluster. If this were true, there would be a higher frequency of areas with either many hits or no hit and a deficiency in the intermediate classes. Table 4 indicates perfect randomness and homogeneity of the area; we have here an instructive illustration of the established fact that to the untrained eye randomness appears as regularity or tendency to cluster.

TABLE 4 EXAMPLE (b): FLYING-BOMB HITS ON LONDON

k	0	1	2	3	4	5 and over
Nk	229	211	93	35	7	1
Np(k; 0.9323)	226.74	211.39	98.54	30.62	7.14	1.57

[14] The figures are taken from R *D.* Clarke, *An application of the Poisson distribution,* Journal of the Institute of Actuaries, vol. 72 (1946), p. 48.

(c) Chromosome interchanges in cells. Irradiation by X-rays produces certain processes m organic cells which we call chromosome interchanges. As long as radiation continues, the probability of such interchanges remains constant, and, according to theory, the numbers *Nk* of cells with exactly k interchanges should follow a Poisson distribution.The theory is also able to predict the dependence of the parameter A on the intensity of radiation, the temperature, etc., but we shall not enter into these details. Table 5 records the result of eleven different series of experiments.[l5] These are arranged according to goodness of fit. The last column indicates the approximate percentage of ideal cases in which chance fluctuations would produce a worse agreement (as judged by the x2-standard). The agreement between theory and observation is striking.

[15] D. G. Catcheside, D. E. Lea, and J. *M.* Thoday, *Types of chromosome structural change induced by the irradiation* of Tradescantia *microspores,* Journal of Genetics, vol. 47 (1945-46), pp. 113-136. Our table is table IX of this paper, except that the X^2-levels were recomputed, using a single degree of freedom.

TABLE 5 EXAMPLE (C): CHROMOSOME INTERCHANGES INDUCED BY X-RAY IRRADIATION

Expt #		Cells with *k* interchanges					
		0	1	2	≥ 3	Total N	X^2 P-Value
1	Observed N_k	753	266	49	5	1073	95
	$N_{Poisson}(k; 0.355)$	*753*	*267*	*47*	*6*		
2	Observed N_k	434	195	44	9	682	85
	$N_{Poisson}(k; 0.456)$	*432*	*197*	*45*	*8*		
3	Observed N_k	280	75	12	1	368	65
	$N_{Poisson}(k; 0.277)$	*279*	*77*	*11*	*1*		
4	Observed N_k	2278	273	15	0	2566	65
	$N_{Poisson}(k; 0.118)$	*2280*	*269*	*16*	*1*		
5	Observed N_k	593	143	20	3	759	45
	$N_{Poisson}(k; 0.253)$	*589*	*149*	*19*	*2*		
6	Observed N_k	639	141	13	0	793	45
	$N_{Poisson}(k; 0.211)$	*642*	*135*	*14*	*1*		
7	Observed N_k	359	109	13	1	482	40
	$N_{Poisson}(k; 0.286)$	*362*	*104*	*15*	*2*		
8	Observed N_k	493	176	26	2	697	35
	$N_{Poisson}(k; 0.336)$	*498*	*167*	*28*	*3*		
9	Observed N_k	793	339	62	5	1199	20
	$N_{Poisson}(k; 0.398)$	*805*	*321*	*64*	*9*		
10	Observed N_k	579	254	47	3	883	20
	$N_{Poisson}(k; 0.405)$	*589*	*239*	*48*	*7*		
11	Observed N_k	444	252	59	1	756	5
	$N_{Poisson}(k; 0.493)$	*461*	*228*	*56*	*11*		

(d) Connections to wrong number. Table 6 shows statistics of telephone connections to a wrong number.[16] A total of N = 267 numbers was observed; N_k indicates how many numbers had exactly k wrong connections. The Poisson distribution *Poisson(k;* 8.74) shows again an excellent fit. (As judged by the X^2-criterion the deviations are near the median value.) In Thorndike's paper the reader will find other telephone statistics following the Poisson law. Sometimes (as with party lines, calls from groups of coin boxes, etc.) there is an obvious interdependence among the events, and the Poisson distribution no longer fits.

TABLE 6 EXAMPLE *(d):* CONNECTIONS TO WRONG NUMBER

k	N_k	$N_{Poisson}(k;\ 8.74)$	k	N_k	$N_{Poisson}(k;\ 8.74)$
0-2	1	2.05	11	20	24.34
3	5	4.76	12	18	17.72
4	11	10.39	13	12	11.92
5	14	18.16	14	7	7.44
6	22	26.45	15	6	4.33
7	43	33.03	≥16	2	4.65
8	31	36.09		267	267.00
9	40	35.04			
10	35	30.63			

[16] The observations are taken from F. Thorndike, *Applications of Poisson's probability summation,* The Bell System Technical Journal, vol. 5 (1926), pp. 604 624. This paper contains a graphical analysis of 32 different statistics.