

(c) Do the results apply to all or year sales forecast. Explain why or why not.

**8.4** When trying to hire managers and executives, companies sometimes verify the academic credentials described by the applicants. One company that performs these checks summarized its findings for a six-month period. Of the 84 applicants whose credentials were checked, 15 lied about having a degree.<sup>7</sup>

- (a) Find the proportion of applicants who lied about having a degree and the standard error.
- (b) Consider these data to be a random sample of credentials from a large collection of similar applicants. Give a 95% confidence interval for the proportion of applicants who lied about having a degree.
- (c) <added by JH> Compare the CI from (b) with the exact Binomial-based CI [obtained via spreadsheet or table... under Resources for Chapter 8]

**8.12** In Exercise 8.1 we found that 11 customers from a random sample of 40 would be willing to buy a software upgrade that costs \$100. If the upgrade is to be profitable, you will need to sell it to more than 20% of your

customers. Do the sample data give good evidence that more than 20% are willing to buy?

- (a) Formulate this problem as a hypothesis test. Give the null and alternative hypotheses. Will you use a one-sided or a two-sided alternative? Why?
- (b) Carry out the significance test. Report the test statistic and the  $P$ -value.
- (c) Should you proceed with plans to produce and market the upgrade?

**8.13** In each of the following cases state whether or not the normal approximation to the binomial should be used for a significance test on a population proportion  $p$ .

- (a)  $n = 10$  and  $H_0: p = 0.6$ .
- (b)  $n = 100$  and  $H_0: p = 0.4$ .
- (c)  $n = 2000$  and  $H_0: p = 0.996$ .
- (d)  $n = 500$  and  $H_0: p = 0.25$ .

- 8.20** A matched pairs experiment compares the taste of instant versus fresh-brewed coffee. Each subject tastes two unmarked cups of coffee, one of each type, in random order and states which he or she prefers. Of the 50 subjects who participate in the study, 19 prefer the instant coffee. Let  $p$  be the probability that a randomly chosen subject prefers freshly brewed coffee to instant coffee. (In practical terms,  $p$  is the proportion of the population who prefer fresh-brewed coffee.)
- (a) Test the claim that a majority of people prefer the taste of fresh-brewed coffee. Report the  $z$  statistic and its  $P$ -value. Is your result significant at the 5% level? What is your practical conclusion?
- (b) Find a 90% confidence interval for  $p$ .

**8.24** You want to estimate the proportion of students at your college or university who are employed for 10 or more hours per week while classes are in session. You plan to present your results by a 95% confidence interval. Using the guessed value  $p^* = 0.40$ , find the sample size required if the interval is to have an approximate margin of error of  $m = 0.06$ .

## "Homegrown" Exercises around M&M Chapter 6

### -1- Help a journalist to be "statistically correct"

age-related prevalence, and conflicting evidence exists in favor of the mortality hypothesis. We compared mortality in a subgroup of 118 opposite-handed twin pairs by counting in how many instances the right-handed twin died first. There was no evidence of differential survival between right-handed and non-right-handed individuals in the entire 1900-1910 cohort. With respect to the number of right-handed twins who died first, there was no material disadvantage among those who were not right-handed. In 60% (95% confidence interval = 49.0-71.5%) of dizygotic pairs, the right-handed twins died first<sup>a</sup>. In 50% of monozygotic pairs<sup>b</sup>, right-handed twins died first. The prevalence of not being right-handed was higher among males (9.2%) than females (6.5%); there was a similar frequency of non-right-handedness in monozygotic (8.0%) and dizygotic (7.8%) twins. We did not find evidence of excess mortality among non-right-handed adult twins in this follow-up study.

Key words: mortality, survival, handedness, twin studies.

- <sup>a</sup> (Approximately) how many dizygotic twin pairs must there have been?
- <sup>b</sup> (Approximately) what is the corresponding CI to accompany the estimate of 50% calculated from monozygotic pairs?
- c Is the 60% significantly different from the 50% at the "conventional" significance level ( $P < 0.05$ )?
- d Calculate the percentage -- of the overall 118 twins pairs -- where the right-handed twin died first, along with an accompanying 95% CI.

### -2- Handedness and Mortality: A Follow-Up Study of Danish Twins Born between 1900 and 1910

Olga Basso, Jørn Olsen, Niels V. Holm, Axel Skytthe, James W. Vaupel, and Kaare Christensen

Epidemiology vol 11 no 5 sept 2000

The declining prevalence of left-handed individuals with increasing age has led to two main avenues of hypotheses; the association is due either (1) to a birth cohort effect and/or an age effect caused by a switch to right-handedness with advancing age or (2) to mortality selection that reduces survival in left-handed individuals, or both. It is uncertain whether a cohort or age effect can explain the decline in

### -1- Variability of, and trends in, proportions

The following data are the proportion of Canadian adults responding YES to the question "Have you yourself smoked any cigarettes in the past week?" in Gallup Polls for the years 1974 to 1985.

1974	52%
1975	47%
1976	---
1977	45%
1978	47%
1979	44%
1980	41%
1981	45%
1982	42%*
1983	41%
1984	39%
1985	39%

--- question not asked;

\* question worded "occasionally or regularly"

Results are based on approximately 1050 personal in-home interviews each year with adults 18 years and over.

- Plot these percentages along with their 95% confidence intervals.
- Is there clear evidence that the trend is downward? To answer this, try to draw a straight line through all (or most of) the confidence intervals and ask can the straight line have a slope of zero i.e. be parallel to the horizontal axis. You might call this a "poor-person's test of trend".

### -11- Triangle Taste test

In its 1974 manual "Laboratory Methods for Sensory Evaluation of Food", Agriculture Canada described tests (the triangle test, the simple paired comparisons test,...) to determine a difference between samples.

"In the triangle test, the panelist receives 3 coded samples and is told that 2 of the samples are the same and 1 is different and is asked to identify the odd sample. This method is very useful in quality control work to ensure that samples from different production lots are the same. It is also used to determine if ingredient substitution or some other change in manufacturing results in a detectable difference in the product. The triangle test is often used for selecting panelists.

Analysis of the results of triangle tests is based on the probability that - IF THERE IS NO DETECTABLE DIFFERENCE - the odd sample will be selected by chance one-third of the time. Tables for rapid analysis of triangle test data are given below. As the number of judgements increases, the percentage of correct responses required for significance decreases. For this reason, when only a small number of panelists are available, they should perform the triangle test more than once in order to obtain more judgements.

The results of a test indicate whether or not there is a detectable difference between the samples. Higher levels of significance do not indicate that the difference is greater but that there is less probability of saying there is a difference when in fact there is none"

Chart: Triangle test difference analysis

[ Table starts at n=7 and ends at n=2000; selected entries shown here ]

Number of Tasters	Number of correct answers necessary to establish level of significance		
	<u>5%</u>	<u>1%</u>	<u>0.1%</u>
7	5	6	7
10	7	8	9
12	8	9	10
30	16	17	19
60	28	30	33
100	43	46	49
1000	363	372	383

## Homegrown Exercises for Chapter 8 [ Inference for proportions ]

### Questions

- a Show how one arrives at the numbers 7, 8 and 9 of correct answers necessary to establish the stated levels of significance for the case of  $n=10$  tasters. Hint: you can work them out from the BINOMDIST function in Excel or [since we are only interested in the principles involved, and not in getting answers correct to several decimal places] you should be able to interpolate them from probability distributions tabulated in the text [the setup here is similar to the therapeutic touch study, but with  $p=1/3$  rather than  $p=1/2$ ].
- b Calculate the exact 90%, 98% and 99.8% 2-sided CI's for the proportions 7/10, 8/10 and 9/10 respectively, and from these limits verify that indeed 7/10, 8/10 and 9/10 are significantly greater than 0.33, at the stated levels of significance. (I am presuming that their  $H_a$  is 1-sided, ie.  $0.33$  vs.  $> 0.33$ )

You can obtain these CI's from the spreadsheet "CI for a proportion", under Resources for Ch 8.

- c Show how one arrives at the numbers 43, 46 and 49 of correct answers necessary to establish the levels of significance for the case of 100 tasters. Hint: you should be able to use a large-sample approximation.
- d How well would this large-sample approximation method have done for the case of  $n=10$ ?
- e If you set the alpha at 0.05 (1-sided), what number of tasters is required to have 80% power to 'detect' a 'shift' from  $H_0: p=1/3$  to (i)  $H_a: p=1/2$  (ii)  $H_a: p=2/3$ ? Use the sample size formula in section 8.1 of the notes.

**Notes:** See worked example 2 in notes on Chapter 8.1. This is a good example where a one-sided alternative is more easily justified, so with  $\alpha = 0.05$  1-sided,  $Z = 1.645$ . Note that power of 80% means that  $\beta = \text{Prob}(\text{failing to reject } H_0) = 1 - \text{power} = 0.2$ , so  $Z = -0.84$ . The  $Z$  is always one-sided, since one cannot be on both sides of  $H_0$  simultaneously!

- f "The triangle test is often used for selecting panelists." -- end of ¶2. Presumably, if one had to choose one of two available panelists, one would ask each to make several judgements. How many judgements would you ask each to make? State any assumptions you make.
- g "When only a small number of panelists are available, they should perform the triangle test more than once in order to obtain more judgements" -- end of ¶3. What scientific objection might one have to this advice?
- h Do you agree with the statement "Higher levels of significance do not indicate that the difference is greater but that there is less probability of saying there is a difference when in fact there is none"--end of ¶4. Why?
- i Explain to somebody who knows little statistics why you think a study with  $n = 6$  tasters would not tell very much. Be statistical, but avoid jargon like 'power' and 'significance' and 'hypothesis'.
- j With a small number of testers, it is possible that, even if a sizeable proportion of the population can correctly taste the , the test of significance will be 'negative'. Suppose that 50% can truly tell the and that 1/3 of the remaining 50% get the test correct by guessing, giving an overall 67% who get the test correct. In this situation, what is the probability that a trial with  $n=12$  will yield a 'positive' (i.e. statistically significant) answer? What if the trial uses  $n=30$ ?  $n=60$ ?

### -12- More U.S. PhD's At McGill than Canadian

MCGILL DAILY, 1993.09.08

McGill professors with a doctorate from Canada are a rare breed when compared to their colleagues who were educated in the United States. According to the 1993-1994 Calendar, in the Faculties of Arts and Science, 42% of professors have American PhDs whereas only 36% have Canadian. This trend has worried some who feel that Canadian PhD graduates are being discriminated against by Canadian universities, and that an education in the United States is unfairly valued over one obtained in Canada.

## Homegrown Exercises for Chapter 8

Letter To MCGILL DAILY September 9, 1993

Considering the numerous issues of real importance that exist, why do you have to invent more? I am referring to your September 8 front page article "More U.S. PhDs at McGill than Canadian," the first sentence of which reads "McGill professors with a doctorate from Canada are a rare breed when compared to their colleagues who were educated in the United States." The second sentence contradicts this; it points out that 36% of Arts and Science professors have Canadian PhDs, vs 42% with U.S. PhDs. This is a deviation of only 6%: roughly the margin or error of Gallop polls. Those who claim that Gallop polls have margin of error of only 4% have forgotten the necessary multiplication by the square root of two. A roughly one to one ration hardly makes Canadian PhDs a "rare breed." In fact, according to your statistics, over one third of our professors... [underlining mine... jh]

### Comment and Questions:

The letter writer asks the Daily «why in your first sentence do you use the phrase "rare compared to their colleagues" when the percentages are 36 and 42? »<sup>1</sup>

We could ask the letter writer «why do you use the phrase "deviation of only 6%" when a simpler "difference of only 6%" would do equally well» and «why complicate things by mentioning Gallop polls and margins of error and the square root of two?»

- a In *one sentence*, explain why one doesn't need inferential statistics here.
- b Also, explain to this writer that if (s)he is going to bring statistical inference about proportions into this, (s)he should get his margin of error correct.