WMS5 Exercises

Make sure you make "Before" and "After" plots of the pdf's

6.1 parts a, b and c , p262
6.13, 6.14 p264
6.19, p270
6.25, 6.26 p 271
6.65, p 286

Homegrown Exercises [more to be added later this week]

Q1 refer again to the exercise we began in class today (Wed 23 May), where

Y ~ N(0,1)

i.e $pdf_Y[y] = (1/\text{sqrt}[2\pi])\ \exp[-y^2/2]$ on (-Infinity, Infinity)

and

$X = Y^2$.

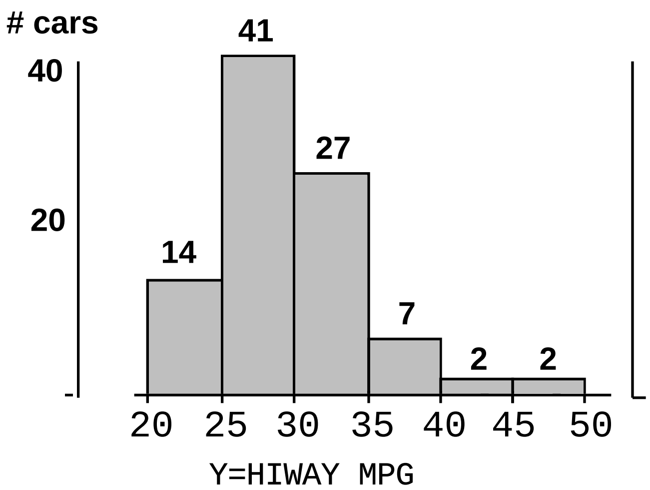
- draw the pdf for Y, the scale for X, and the "backtrack" from X to Y.
- derive the pdf, $pdf_X[x]$ of X at selected X values, say X=0(1)10.

(Note that you will have to first "pour" the density from the Y≥0 region onto the X ≥ 0 region. and then add to it the probability from the Y < 0 region.)

If you prefer, you can do the calculations/graphs by Excel, (it has the Gaussian pdf built in -- or you can write the formula yourself)

- From these numerical calculations, work out the $pdf_X[x]$ for any value x.
  and check that the integral of $pdf_X[x]$ is indeed unity.
- Do you recognize the distribution of X ? What are its parameters?

Q2 **Y = fuel economy, measured in miles per gallon (mpg) on the highway, of 1993 model automobiles.. (93 cars in all)**



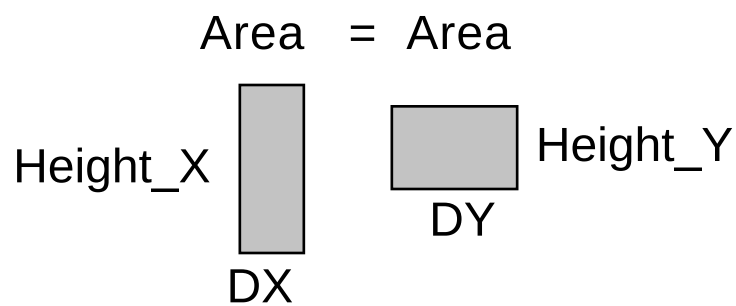
a The scale on the left vertical axis is absolute numbers of cars. Add a <u>density</u> scale on the right vertical axis, making sure that the total probability (total <u>area</u> of 6 rectangles) is 1. Freedman says it is useful to think of density as "proportion per unit of MPG"

b Change the variable of interest to X = the number of gallons of gas to go 200 miles

i.e X = 200 / Y

Draw the histogram for X, making sure that the total area sums to 1. Normally, we would choose equal width-intervals for X (the new variable of interest). in this case, since we don't have access to the raw data, make an exception and let the boundaries for the Y intervals dictate the boundaries of the intervals for X i.e. simply transfer the intervals for Y into their equivalents (now of unequal widths) for X.

A good way to be sure that the heights of the new rectangles sitting over the new intervals are correct is to use the "law of conservation of probability" ie

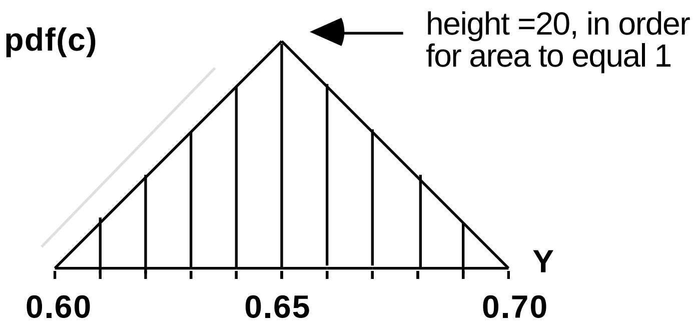


$$\text{Height\_New} = \text{Height\_Old} \bullet \frac{\text{D\_Old}}{\text{D\_New}}$$

Q3 Given:

C = What Canadian $ is worth, in $US over the last year ($pdf_Y(y)$ say)

Say distribution (pdf) of C is given by..



U = what a US$ is worth, in $ Canadian, in same period.

i.e., U = 1/C

a Compute the pdf for U via the Method of Transformations (i.e. "pdf directly to pdf")

The range of U is 1/0.7 = 1.43 to 1.67, a total distance of 24 cents, versus only 0.10 for C

So the pdf for U will have to be lower so that the total area associated with it equals unity.

To save you time, equations that describe the pdf of C are

$$pdf_C(c) = \begin{cases} 400(c - 0.60) & 0.60 \le c \le 0.65; \\ 400(0.70 - c) & 0.65 \le c \le 0.70. \end{cases}$$

b Compute the pdf for U via the Method of Distribution Functions (i.e. "pdf from cdf from cdf")

$$cdf_C(c) = \begin{cases} 8(5c - 3)^2 & 0.60 \le c \le 0.65; \\ 280c - 200c^2 - 97 & 0.65 \le c \le 0.70. \end{cases}$$

The pdf for U should look like..