INTERPRETATIONS OF PROBABILITY (§1.2 of **MRT2**)

At the mathematical level, there is hardly any disagreement about the foundations of probability or about its mathematical consequences. The foundation in set theory was laid in 1933 by the great Russian probabilist, A. Kolmogorov, still an active research worker in 1969. At the level of interpretation and use, there are two extreme positions that are often adopted and, of course, many positions in between.

1) ***Objective****.* This position holds that probability is applicable only to events that can be repeated over and over under much the same conditions. Thus the objectivist is happy to talk about probabilities in connection with the tossing of a coin or the manufacture of a mass-produced item. He can readily think of many light bulbs being produced, and of the probability of a good light bulb as the long-run ratio of the number of good bulbs to the total number produced. But he draws the line at unique events. For example, he would not care to talk about the probability that Romulus founded Rome, or that Chile and Argentina would unite to become a single country in the next ten years. Thus a large class of problems is set aside by the objectivist as not appropriate for the application of probability, because there is no long-run ratio in view. Furthermore, the objectivist likes to make interpretations only from repeated events.

2) ***Personal****.* The personalist regards probability as a measure of personal belief in a particular proposition, such as the proposition that it will rain tomorrow. This school of thought believes that different "reasonable" individuals only differ in their degrees of belief, even when offered the same evidence; and their personal probabilities for the same event may differ. The personalist will apply probability to all the problems an objectivist studies, and to many more. For example, at least in principle, the personalist would take the Romulus question in his stride. The personalist also has available some additional techniques; in particular, he may have more use for Bayes's Theorem, treated in Chapter 4, than the objectivist. On the other hand, when the amount of data is large, the objectivist and the personalist usually get similar answers.

A beginner would be unwise to try to decide at once where he fits in with respect to these two views. Furthermore, the last word is never said on such matters because new schools of thought arise. But the distinction between probability as a long-run relative frequency and probability as a measure of belief is one that he may wish to reconsider from time to time as he understands the issues better.

OTHER TEXTBOOKS' VIEWS OF PROBABILITY

M&M3 [Chapter 4: Probability: The Study of Randomness]

"Probability is the branch of mathematics that descibes the pattern of chance outcomes. Probability is the topic of this chapter, but not for its own sake. We will concentrate on the ideas of probability that we need to go more deeply into statistics" M&M3 Preface to Chapter 4

"The reasoning of statistical inference rests on asking "How often would this (statistical) method (procedure) give a correct answer if I used it many times?" When we produce data by random sampling or randomized comparative experiments, the laws of probability answer the question "What would happen if we did this many times?" M&M3 p290

"The language of probability..
Random" in statsitics is not a synonym for "haphazard" but a description of a kind of order that emerges only in the long run. (...) In the long run, the proportion of tosses of a (fair) coin that give a head is 0.5. This is the intuitive idea of probability. Probability 0.5 means "occurs half the time in a very large number of trials" M&M3 p290

We might suspect that a coin has probability 0.5 of coming up heads just because the coin has two sides*. As exercises 4.1 and 4.2 illustrate**, such suspicions are not always correct. The idea of probability is empirical. That is, it is based on observation rather than theorizing. Probability describes what happens in very many trials, and we must actually observe many trials to pin down a probability. M&M3 p290
(** exercises reproduced in assignment 2)

*Mosteller et al. (p113-) use an even better example (dropping ordinary thumbtacks onto a hard surface, where they bounce before coming to rest, with the sharp end pointing up or down. They also mention the example of the mathematician D'Alembert in connection with the outcome of tossing two fair coins. He reasoned that there were were three possible ways the coins could fall (a) both heads (b) one head and one tail, and (c) both tails. He thought that these three outcomes were equally likely.

"Randomness and Probability

We call a phenomenon **random** if individual outcomes are uncertain but there is nevertheless a regular distribution of outcomes in a large number of repetitions.

The **probability** of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions. That is, proability is long-term relative frequency." M&M3 p291

FPPA2 [Chapter 13: What are the Chances?]

FPPA use the term "chance" † instead of "probability" for much of their book, no doubt influenced by the words and work of a person whom they they call one of the "great masters" -- Ahraham de Moivre (1667-1754) and his book *The Doctrine of Chances**

FPPA2 explain that..

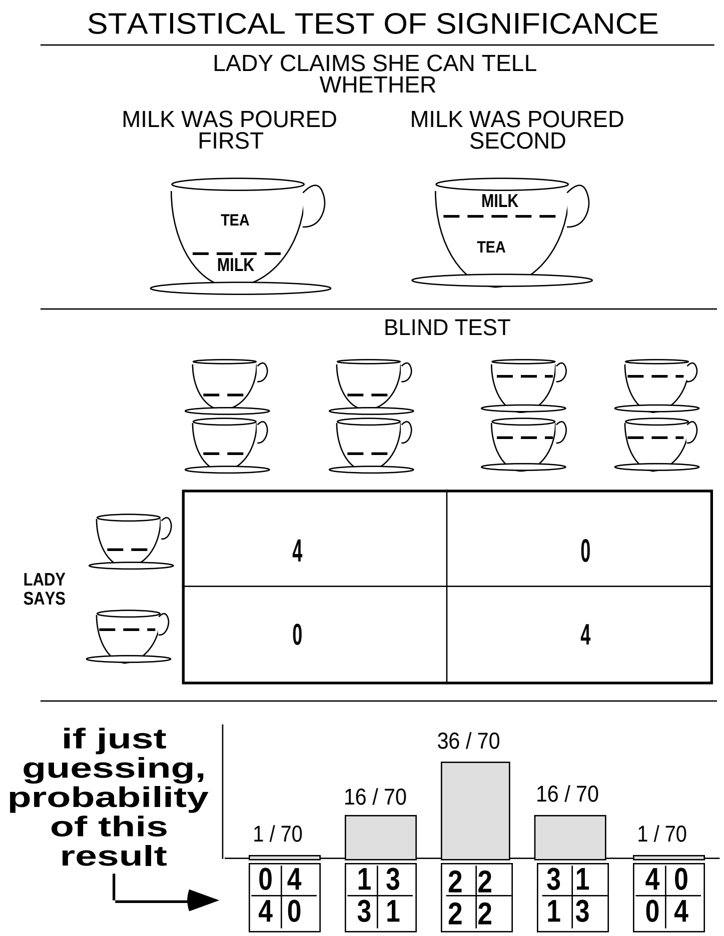
> "The ***chance*** of something gives the percentage of time it is expected to happen, when the basic process is done over and over again, independently, and under the same conditions."
> FPPA2 p 208

* FPPA2 reproduce two extracts from The Doctrine of Chances. These can be found elsewhere on the web site.

A Tip from the Great Master: *"Some of the Problems about Chance have a great appearance of Simplicity, the Mind is easily drawn into a belief, that their Solution may be attained by the mere Strength of Natural good Sense; which generally proving otherwise and the Mistakes occasioned thereby being not infrequent, 'tis presumed that a Book of this Kind, which teaches to distinguish Truth from what seems nearly to resemble it, will be looked upon as a help to good Reasoning" (FPPA2p223)*

† MRT2, p 19, introduce probability as "a measure of chance"

EXAMPLE 1 (see R. A Fisher, Design of Experiments Chapter 2)



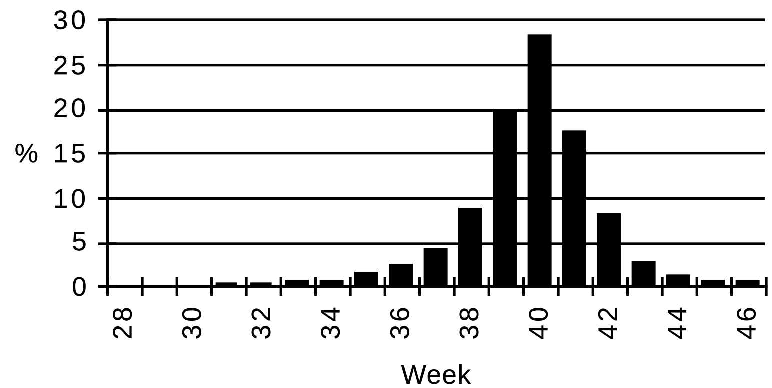
EXAMPLE 2

In 1949 a divorce case was heard in which the sole evidence of adultery was that a baby was born almost 50 weeks after the husband had gone abroad on military service.

[Preston-Jones vs Preston-Jones, English House of Lords]

To quote the court "The appeal judges agreed that the limit of credibility had to be drawn somewhere, but on medical evidence 349 (days) while improbable, was scientifically possible." So the appeal failed.

Pregnancy Duration: 17000 cases > 27 weeks
(quoted in Guttmacher's book)



In U.S., [Lockwood vs Lockwood, 19??], a 355-day pregnancy was found to be 'legitimate'.

<u>Other Examples:</u>

- Quality Control (it has given us terminology)
- Taste-tests (see exercises)
- Adding water to milk.. see M&M Example 6.6 p448
- Water divining.. see M&M exercise 6.44 p471
- Randomness of U.S. Draft Lottery of 1970.. see M&M Example 6.6 p105-107, and 447-
- Births in New York City after the "Great Blackout"
- John Arbuthnot's "argument for divine providence"
- US Presidential elections: Taller vs. Shorter Candidate.

EXAMPLE 3: Is woman a Haemophilia Carrier?

From what we already knew about her her brothers:

Probability [**WOMAN** is Carrier] = 0.5

New Data: Her **Son** is Normal (NL) .

In light of new data..

Probability[**WOMAN** is Carrier, given her son is NL] = **??**

EXAMPLE 4.

Of 14 boys followed up for a median of 5 1/2 years after chemotherapy for leukemia, none had abnormal testicular function (i.e., the abnormality rate was 0/14).1 With what risk, if any, of testicular dysfunction might these results be compatible?

EXAMPLE 5 [La Presse, Montréal, 1993]

L'Institut Gallup a demandé récemment à un échantillon représentatif de la population canadienne d'évaluer la manière dont le gouvernement fédéral faisait face à divers problèmes économiques et général. Pour 59 pour cent des répondants, les libéraux n'accomplissent pas un travail efficace dans ce domaine, tandis que 30 pour cent se déclarent de l'avis contraire et que onze pour cent ne formulent aucune opinion.

La même question a été posée par Gallup à 16 reprises entre 1973 et 1990, et ce ne n'est qu'une seule fois, en 1973, que la proportion des Canadiens qui se disaient insatisfaits de la façon dont le gouvernement gérait l'économie a été inférieure à 50 pour cent.

Les conclusions du sondage se fondent sur **1009** interviews effectuées entre le 2 et le 9 mai 1994 auprès de Canadiens âgés de 18 ans et plus. **Un échantillon de cette ampleur donne des résultats exacts à 3,1 p.c., près dans 19 cas sur 20.** La marge d'erreur est plus forte pour les régions, par suite de l'importance moidre de l'échantillonnage; par exemple, les **272** interviews effectuées au Québec ont engendré une marge d'erreur de **6 p.c**. dans **19 cas sur 20**.

EXAMPLE 6 The Nielsen system for TV ratings (U.S.A.)

"...Nielsen uses a device that, at one minute intervals, checks to see if the TV set is on or off and to which channel it is tuned. That information is periodically retrieved via a special telephone line and fed into the Nielsen computer center in Dunedin, Florida.

With these two samplings, Nielsen can provide a statistical estimate of the number of homes tuned in to a given program. A rating of 20, for instance, means that 20 percent, or 16 million of the 80 million households, were tuned in.

To answer the criticism that 1,200 or 1,500 are hardly representative of 80 million homes or 220 million people, Nielsen offers this analogy:

Mix together 70,000 white beans and 30,000 red beans and then scoop out a sample of 1000. the mathematical odds are that the number of red beans will be between 270 and 330 or 27 to 33 percent of the sample, which translates to a "rating" of 30, plus or minus three, with a 20-to-1 assurance of statistical reliability. The basic statistical law wouldn't change even if the sampling came from 80 million beans rather than just 100,000."

EXAMPLE 7

The status of 112 liveborn children whose mothers had been immunized against rubella was studied to assess the risks of gestational exposure to the vaccine. None of the infants born (0/112) had any congenital malformations associated with congenital rubella. What is the maximum malformation risk compatible with finding none of 112 infants with defects in a single study?

DEFINITIONS • "Experiment" • "Simple" Event • Sample Space • Event • Probability Axioms

Text	"Experiment"...
WMS5	"the process by which an observation is made"
MRT2 (p 18-19)	"any act that can be repeated under given conditions"
M&M3 (p 234)	"an **observational study** observes individuals and measures variables of interest but does not attempt to influence the responses An **experiment**, on the other hand, deliberately imposes some treatment on individuals in order to observe their responses"
FPPA2 (p 11)	"In a **controlled experiment** (the title of the authors' chapter 1, with the Salk vaccine field trial as their example), the investigators decide who will be in the treatment group and who will be in the control group. By contrast in an **observational study** (the title of the authors' chapter 2*, with studies of the effect of cigarette smoking on humans as their first example) it is the subjects who assign themselves to the different groups: the investigators just watch what happens." *Introduced with the quote, from Sir Ronald Fisher *"That's not an experiment you have there, that's an experience"*
OED3 (1944)	1. The action of trying anything; a test, trial; 2. A procedure adopted in uncertainty whether it will answer the purpose 3. An action or operation undertaken in order to discover something unknown, to test a hypothesis, or establish or illustrate some known truth

Text	"Event"...
WMS5	"**Outcome** of an experiment" (later: "any subset of S")
MRT2 (p 21)	"Sometimes it is convenient to regard a **set of outcomes** as a single *event*"
M&M3 (p 234)	"An event is an **outcome or set of outcomes** of a random phenomenon. That is, an event is a subset of the sample space"
	"Simple / Compound Event"...
WMS2	Simple: "An event that **cannot be decomposed**" Otherwise: "compound" event "Each simple event corresponds to 1 and only 1 **sample point"**
	"Sample Space"...
WMS5	Sample space S associated with an experiment is the "**set of all possible sample points (outcomes)**"
M&M3	"The sample space S of a random phenomenon is the **set of all possible outcomes**"
MRT2	"A sample space S of an experiment is the set of elements such that any one outcome of the experiment corresponds to exactly one element in the set. An element in a sample space is called a sample point. " "Depending on *interest*, there can be different S's: that's why we talk about "a" rather than "the" sample space of the experiment" e.g. 2 coins tossed, each produces a H or a T *interest*: sequence # of H's Coins Alike/Different? S={HH,HT,TH,TT} S={2,1,0} S={Alike,Different}
FPPA2	"When trying to figure chances, sometimes helpful to list all the possible ways that a chance process can turn out" [They don't say so, but their "list" is a sample space]

AXIOMS OF PROBABILITY

WMS2 - see DEFINITION 2.6 page 27

MRT2 - more in words...

I	*Positiveness*	The probability assigned to each event is positive or zero.
II	*Certainty*	The probability of the entire sample space is 1.
III	*Unions*	If A and B are mutually exclusive (non-overlapping / "disjoint") events, then

P(A or B) = P(A) + P(B)

M&M3 add 1 more (derivable from I-III, but very useful)

IV *Complement* P("not" A) = 1 – P(A)

FPPA2 - Two basic facts will now be stated...

> Chances are between 0% and 100%

> The chance of something equals 100% minus the chance of the opposite thing.

Definition: Two things are mutually exclusive when the occurrence of one prevents the occurrence of the other.

> To find the chance that at least one of two things will happen, check to see if they are mutually exclusive. If they are, add the chances.

Examples

• From M&M3

Describe a sample space S for the following random phenomena.

(a) Choose a student in your class at random. Ask how much time that student spent studying in the last 24 hours.

(b) Ditto, but ask how much cash that student is carrying.

(c) The US Physicians' Health Study asked 11,000 physicians to take and aspirin evert other day and observed how many of them had a hear attack in a five year period.

(d) In a test of a new package design, you drop a carton of a dozen eggs from a height of 1 metre and count the number of broken eggs

• From www.stat.umn.edu/~luke/classes/3091-3/)

Comment. on the probabilistic weather forecasts given by these three TV stations..

Channel	4	5	11
Prob(Rain)	0.1	0.2	0.3
Prob(Overcast)	0.4	0.3	0.5
Prob(Sunny)	0.5	0.6	0.2

• From MRT2 p27 [very helpful to distinguish elements)

An experiment consists of throwing two ordinary six-sided dice (one red, one clear) and observing the numbers of dotds on their upper faces

Clear die outcome (c)

red die outcome (r)	**1**	**2**	**3**	**4**	**5**	**6**
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Events		
	(i) a double	prob = ??
	(ii) $c \geq r$ by at least 3	prob = ??
	(iii) sum equals 10	prob = ??

E.g. 1. **Tea-tasting** (similar structure to exercise 2.14)

• **2 cups**, 1 with milk added first, 1 with milk added second
Subject told that there is 1 of each.

If unable to tell, but just guessing, what is chance of correctly identifying which is which?

		1st cup	2nd cup
w.l.o.g.	TRUTH	first	second
possibilities			
1	Guess	first	second (D)
2	Guess	second	first (D)

(D) By Default

• **3 cups,** 1 with milk added first, 2 with milk added second
Subject told mix is 2 & 1 (but not which there are 2 of)

		1st cup	2nd cup	3rd cup
w.l.o.g.	TRUTH:	first	second	second
possibilities				
1	Guess	first	second	second (D)
2	Guess	second	first	second (D)
3	Guess	second	second	first (D)

• **4 cups**, 2 with milk added first, 2 with milk added second

Subject told that mix is 2 and 2

		1st cup	2nd cup	3rd cup	4th cup
w.l.o.g.	T->	first	second	second	first
possibilities					
1	G	first	first	second (D)	second (D)
2	G	first	second	first	second (D)
3	G	first	second	second	first (D)
4	G	second	first	first	second (D)
5	G	second	first	second	first (D)
6	G	second	second	first (D)	first (D)

E.g. 2. **Hat problem** (cf.variation in text example 2.3)

"3/4ths of the time, 2 of the players will have hats of same color and the third player's hat will be the opposite color"