Theory

Simulations

AfterMath / Fisher / $z \rightarrow t$

Messages

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Student's z, t, and s: What if Gosset had R?

James A. Hanley¹ Marilyse Julien² Erica E. M. Moodie¹

¹Department of Epidemiology, Biostatistics and Occupational Health, ²Department of Mathematics and Statistics, McGill University

Gosset Centenary Session organized by Irish Statistical Association XXIVth International Biometric Conference

Dublin, 2008.07.16

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Introduction	Theory	Simulations	AfterMath / Fisher / $z \rightarrow t$	Mes
Volum	E VI	MARCH,	1908	No. 1

BIOMETRIKA.

THE PROBABLE ERROR OF A MEAN.

BY STUDENT.

William Sealy Gosset, 1876-1937

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"STUDENT"

Annals of Eugenics 1939



The untimely death of W. S. Gosset (...) has taken one of the most original minds in contemporary science.

Without being a professional mathematician, he first published, in 1908, a fundamentally new approach to the classical problem of the theory of errors, the consequences of which are only still gradually coming to be appreciated in the many fields of work to which it is applicable.

The story of this advance is as instructive as it is interesting.

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Messages

MR. W. S. GOSSET Obituary, The Times, 1937

THE INTERPRETATION OF STATISTICS

"E.S.B." writes:-

My friend of 30 years, William Sealy Gosset, who died suddenly from a heart attack on Saturday, at the age of 61 years, was known to statisticians and economists all over the world by his pseudonym "Student," under which he was a frequent contributor to many journals. He was one of a new generation of mathematicians who were founders of theories now generally accepted for the interpretation of industrial and other statistics.

E.S.B.: Edwin Sloper Beaven (1857-1941): one of leading breeders of barley in first half of 20th century. 1894; purchased 4 acres of land at Boreham just outside Warminster & began to carry out experimental trials of barley. Associated with Arthur Guinness, Son & Co who took over his maltings and trial grounds after his death.

The eldest son of Colonel Frederic Gosset, R.E., of Watlington, Oxon, he was born on June 13, 1876. He was a scholar of Winchester where he was in the shooting VIII, and went up to Oxford as a scholar of New College and obtained first classes in mathematical moderations in 1897 and in natural science (chemistry) in 1899. He was one of the early pupils of the late Professor Karl Pearson at the Galton Eugenics Laboratory, University College, London. Over 30 years ago Gosset became chief statistician to Arthur Guinness, Son and Company, in Dublin, and was guite recently appointed head of their scientific staff. He was much beloved by all those with whom he worked and by a select circle of professional and personal friends, who revered him as one of the most modest, gentle, and brave of men, unconventional, yet abundantly tolerant in all his thoughts and ways. Also he loved sailing and fishing, and invented an angler's self-controlled craft described in the *Field* of March 28, 1936. His widow is a sister of Miss Phillpotts, for many years Principal of Girton College, Cambridge. ◆□▶ ◆□▶ ▲□▶ ▲□▶ □ のQ@

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March 2, 1938

Dear Mrs Gosset,

I am preparing an obituary of Mr Gosset for the Annals of Eugenics, and should be glad to publish therewith a photograph of him if you could let me have one which you would **guides** suitable for this purpose. Yours sincerely,

http://digital.library.adelaide.edu.au/coll/special//fisher/

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march 8: 1938

DAVAN HOLLOW DENHAM BUCKS

Dear D' Fisher Thank you for your letter. Unfortunately my husband would never consent to be photographed during recent years. I have had some supposites enlarged, but They are not sharp enough to be of any use for reproduction. I have a fairly good shoto of him taken about 1908, but I suppose that would not do? my husband's brother toto lives in Calcutta, sent me

a cutting from an Indian paper, giving your speech at the Conpress, + it was a great pleasure to me to read in it your tribule to Students' work.

Jours Succerely Marjory Gosset

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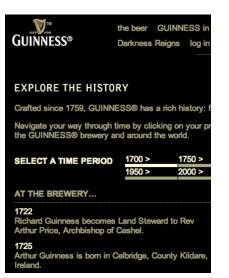
The Gossets... [from Burke: The Landed Gentry]

- Of "Norman Extraction"
- **Coat of arms:** D'àgur, à un annulet d'or, et trois Goussés de fèves feuillées et tigées, et rangées, en pairle de même; au chef d'argent, chargé d'une aiglette de sable.
- 1555: Adopted Protestant faith \rightarrow name removed from roll of nobles.
- 1685: Revocation of Edict of Nantes → Jean Gosset, a Huguenot, moved to Island of Jersey.
- Some of Jean Gosset's family settled in England.

http://www.geocities.com/Heartland/Hollow/9076/FOGp1c1.html

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http://www.guinness.com/



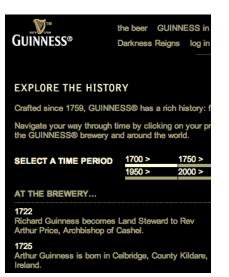
1893:

T. B. Case becomes the first university science graduate to be appointed at the GUINNESS brewery.

It heralds the beginning of 'scientific brewing' at St. James's Gate.

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http://www.guinness.com/

1936 The first GUINNESS® brewery overseas is built at Park Royal, London. William Sealy Gossett, the father of modern statistics, is appointed Head Brewer.

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Lead up to 1908 article from appreciation by Egon S Pearson, 1939

- 1899 Hired as a staff scientist by Guinness (Dublin)
- 1904 "The Application of the 'Law of Error' to the work of the Brewery"
 - Airy: Theory of Errors Merriman: A textbook of Least Squares
- '06-'07 At Karl Pearson's Biometric Laboratory in London.
 - 1907 Paper on sampling error involved in counting yeast cells.
 - 1908 Papers on P.E. of *mean* and of *correlation coefficient*.

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Gosset's introduction to his paper

"Usual method of determining the probability that μ lies within a given distance of \bar{x} , is to assume ..."

$$\mu \sim N(\bar{x}, s/\sqrt{n}).$$

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But, with smaller n, the value of s "becomes itself subject to increasing error."

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Forced to "judge of the uncertainty of the results from a small sample, which itself affords the only indication of the variability."

The method of using the normal curve is only trustworthy when the sample is "large," no one has yet told us very clearly where the limit between "large" and "small" samples is to be drawn.

Aim ...

"to determine the point at which we may use the (Normal) probability integral in judging of the significance of the mean ..., and to furnish alternative tables when [n] too few."

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Sampling distributions studied

$$ar{x}=rac{\sum x}{n}$$
; $s^2=rac{\sum (x-ar{x})^2}{n}.$

"when you only have quite small numbers, I think the formula with the divisor of n - 1 we used to use is better"

... Gosset letter to Dublin colleague, May 1907

Doesn't matter, "because only naughty brewers take *n* so small that the difference is not of the order of the probable error!"

... Karl Pearson to Gosset, 1912

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Three steps to the distribution of z

Section I

- Derived first 4 moments of s^2 .
- Found they matched those from curve of Pearson's type III.
- "it is probable that that curve found represents the theoretical distribution of s²." Thus, "although we have no actual proof, we shall assume it to do so in what follows."

Section II

- "No kind of correlation" between \bar{x} and s
- His proof is incomplete: see ARTICLE in *The American Statistician*.

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Section III

- Derives the pdf of z:
 - joint distribution of $\{\bar{x}, s\}$
 - transforms to that of $\{z, s\}$,
 - integrates over s to obtain $pdf(z) \propto (1 + z^2)^{-n/2}$.

Sections IV and V

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Sections IV and V



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Section VI: "Practical test of foregoing equations."

[pdf's of s and z "are compared with some actual distributions"]

Before I had succeeded in solving my problem analytically, I had endeavoured to do so empirically.

The material used was a correlation table containing the height and left middle finger measurements of 3000 criminals, from a paper by W. R. Macdonell (*Biometrika*, Vol. I, p. 219).

The measurements were written out on 3000 pieces of cardboard, which were then very thoroughly shuffled and drawn at random.

As each card was drawn its numbers were written down in a book, which thus contains the measurements of 3000 criminals in a random order.

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Finally, each consecutive set of 4 was taken as a sample – 750 in all – and the mean, standard deviation, and correlation of each sample determined.

The difference between the mean of each sample and the mean of the population was then divided by the standard deviation of the sample, giving us the z of Section III.

This provides us with two sets of 750 standard deviations and two sets of 750 z's on which to test the theoretical results arrived at.

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Inside cover of one of Gosset's notebooks...



photo courtesy of Elizabeth Turner LSHTM, and UCL archives.

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Macdonell's data

See HANDOUT & WEBSITE

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- Reproduced means and sd's reported by Macdonell.
- Repeated Gosset's procedure to create 750 samples.
- Occasionally, all 4 persons from same 1" bin $\rightarrow s = 0$. Replaced $z = \pm \infty$ by \pm largest observed |z|.
- X^2 goodness of fit statistic for 750 s/σ , and 750 z values.
- Repeated procedure 100 times:- 100 X² values :- check repeatability of Gosset's X² statistics; cards sufficiently shuffled?
- Single set of 75,000 samples of size 4, *sampled with replacement*, and with *Scotland Yard precision* (1/8 of 1"). How much more smooth/accurate might Gosset's empirical frequency distribution of *s* have been?

Our simulations, 100 years later

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- Repeated procedure 100 times:- 100 X² values :check repeatability of Gosset's X² statistics; cards sufficiently shuffled?
- Single set of 75,000 samples of size 4, *sampled with replacement*, and with *Scotland Yard precision* (1/8 of 1"). How much more smooth/accurate might Gosset's empirical frequency distribution of *s* have been?

- Reproduced means and sd's reported by Macdonell.
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Messages

RESULTS: his and ours

Shuffling:

	No. samples/750 with <i>s</i> = 0								
	0	- 1	2	3	4	5		All	
Ours:	21	41	17			1		100	
Gosset's:				1				1	

Gosset's double precautions – very thorough shuffling *and* drawing cards at random – appear to have worked.

Unlike the 1970 U.S. draft lottery for military service in Vietnam

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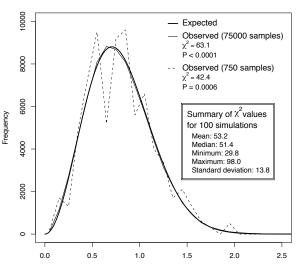
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Distribution of s/σ

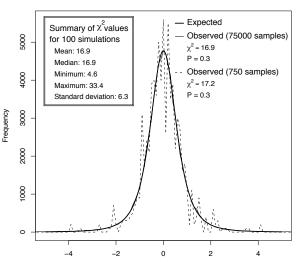


Scale of standard deviation

Dotted line: Sample statistics obtained from one set of 750 random samples generated by Gosset's procedure. Inset: distr'n of 100 X^2 statistics (18 intervals). Thin solid line: distr'n of statistics obtained from 75,000 samples of size 4 sampled with replacement from 3000 heights recorded to nearest 1/8".

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Distribution of z



Dotted line: Sample statistics obtained from one set of 750 random samples generated by Gosset's procedure. Inset: distribution of 100 X^2 statistics (15 intervals). Thin solid line: distr'n of statistics obtained from 75,000 samples of size 4 sampled with replacement from 3000 heights recorded to nearest 1/8".



If Gosset had R:

"Agreement between observed and expected frequencies of the 750 s/σ 's was not good". He attributed this to coarse scale of s.

Distribution of our 75,000 s/σ values also shows pattern of large deviations similar to those in table on p. 15 of his paper.

Scotland Yard precision and today's computing power would have left Gosset in no doubt that the distribution of s which he "assumed" was correct was in fact correct.

Grouping had not had so much effect on distr'n of z's: "close correspondence between the theory and the actual result."

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FOR...

- Description of remainder of 1908 article
- · Early extra-mural use of Gosset's distribution
- Fisher's geometric vision
- Fisher and Gosset , and transition $z \rightarrow t$

SEE..

- SLIDES FROM LONGER VERSION OF TALK
- ARTICLE in The American Statistician

Theory

Simulations

AfterMath / Fisher / $z \rightarrow t$

Messages

Triumphator A ser 43219



http://www.calculators.szrek.com/

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To students of statistics in 2008 ... (I)

Fisher1939:

- "of (Gosset's) personal characteristics, the most obvious were a clear head, and a practice of forming independent judgements."
- The other was the importance of his work environment: "one immense advantage that Gosset possessed was the concern with, and responsibility for, the practical interpretation of experimental data."

Gosset stayed very close to these data.

We should too!

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To students of statistics in 2008 ... (II)

Compared with what Gosset could do, today we can run much more extensive simulations to test our new methods.

Which pseudo-random observations are more appropriate: those from perfectly behaved theoretical populations, or those from real datasets, such as Macdonell's?

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Today's students – and teachers – would do well to heed E.S. Pearson's 1939 advice regarding writing and communication.

E.S. Pearson on Gosset's 'P.E. of Mean' paper...

"It is a paper to which I think all research students in statistics might well be directed, particularly before they attempt to put together their own first paper."

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FUNDING / CO-ORDINATES

Natural Sciences and Engineering Research Council of Canada

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